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Permit trading with flow pollution and stock pollution

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ABSTRACT

Water pollutants are non-uniformly mixed across space and different in the persistence of environmental damage. Their marginal environmental damage could vary to a large degree with emission locations. The efficient pollution control should integrate heterogeneity in the marginal environmental damage caused by pollutants. However, little literature on permit trading has considered the scale and the persistence of environmental damage in the meantime. This paper designs proper trading ratios in permit trading to alleviate environmental damage from both aspects. It finds that it is not only important but also indispensable to incorporate the decay rate of pollutant, the discount rate, and the initial pollutant stock into permit trading, so as to achieve the efficient pollution control.

1. Introduction

The success of permit trading in air pollution inspires people to extend its application to water pollution. Compared with air pollution, Fisher-Vanden and Olmstead (2013) have summarized five challenges for permit trading to achieve the same success in water pollution. The first and foremost one is non-uniform mixing. Water pollutants are not uniformly mixed, so that the marginal environmental damage of water pollution may differ to a large degree with emission locations (Fisher-Vanden and Olmstead, 2013). Considering the characteristics of receiving waters and pollutants, water pollution can be categorized into flow pollution and stock pollution.

The main difference between flow pollution and stock pollution is the persistence of their environmental damage. Flow pollution is caused by pollutant flows. Its damage will cease immediately once the pollutant flow is gone. Stock pollution is caused by pollutant stock. Its damage will sustain until all the pollutant completely decays. Chemicals, organisms and nutrients could lead to both flow pollution and stock pollution, depending on their emission locations and their decay rates. The examples are widespread and include mercury contamination in waters of California which can be traced back to the Gold Rush; water deterioration in the Grand River and the Lake Michigan (Schrauben, 2010); and eutrophication in the Mississippi River and the Gulf of Mexico.[1] In most cases, river pollution belongs to flow pollution, since pollutants are easily taken away by water flows,

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[1] Flow pollution and stock pollution coexist in air pollution, too. Take SO_2 and NO_x as an example, they are controlled in the Acid Rain Program from the power sector. These pollutants cause flow pollution in the atmosphere, but decay quickly there. They also cause stock pollution in the way of acidification of soil and water bodies.The influence of CO_2 as a Greenhouse gas can also be considered as stock pollution because it persists for centuries in the atmosphere.

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and their influence on a specific location along a river is temporary (Lieb, 2004).[2] Lake pollution, on the other hand, can be considered as stock pollution, as pollutants may accumulate and affect the environment over time.

Back to the non-uniform mixing problem, location-related trading ratios have been developed, in the spirit of exchange rates, to account for the impact of emissions from different locations. Actually, Montgomery (1972) in his seminal paper on permit markets has already discussed the idea of using a trading-ratio system in permit trading. More recently, Hung and Shaw (2005) design a trading-ratio system based on the transfer coefficients among different areas. At the same time, Farrow et al. (2005) develop another trading-ratio system based on the momentary damage caused by emissions from different locations. This damage-based trading-ratio system considers more than the transfer coefficients, which also involves demographic and economic factors. Holland and Yates (2015) further improve this trading-ratio system when environmental damage is uncertain.

However, the current design of trading ratios has not yet considered heterogeneity in the persistence of environmental damage caused by pollutants. The designs of Hung and Shaw (2005) and Farrow et al. (2005) are applicable only if there is no difference in the persistence of environmental damage, that is, all pollution belongs to either flow pollution or stock pollution (with the same decaying rate). Muller and Mendelsohn (2009) provide an example of using a damage-based trading-ratio system to improve the SO_2 allowance trading program for power plants.[3] Regarding permit trading in water pollution (WQT), an increasing number of studies consider the possibility of point-nonpoint trading. Horan and Shortle (2011) suggest to incorporate uncertainties related to nonpoint pollution in trading ratios, and Nguyen et al. (2013) simulate point-nonpoint trading under different scales of uncertainties. But these analyses of the trading ratios do not consider heterogeneity in the persistence of environmental damage, either. Particularly for those using the Huang and Shaw trading ratios, it does not seem that flow pollution and stock pollution have been distinguished (e.g. Mesbah et al. (2009); Obropta and Rusciano, 2006; Roberts and Clark, 2008; Mesbah et al. (2009)).

Due to the characteristics of flow pollution and stock pollution, they require different analytical models (Hoel and Karp, 2001). Flow pollution fits a static model because it is only influenced by the pollutant in current period. Stock pollution, on the other hand, needs a dynamic model to account for accumulation of the pollutant and its impact on the environment. Although there are dynamic analysis on the optimal path of abatement to control stock pollution and also on environment dynamics which are influenced by stochastic factors and people behaviors (e.g. Dechert and O'Donnell, 2006; Iwasa et al. (2007); Laukkanen and Huhtala, 2008; Hediger, 2009), no one has incorporated permit trading to achieve the cost-effective outcome of stock pollution control to our knowledge.[4] The research on WQT or permit trading, on the other hand, has not considered the link between the current and the future environmental damage of stock pollution. In order to solve the non-uniform mixing problem of water pollution, it is important to consider heterogeneity not only in the scale but also in the persistence of environmental damage in permit trading.

There is only one exception which notice heterogeneity in the persistence of environmental damage in permit trading. Leiby and Rubin (2001) derive an intertemporal trading-ratio system under both flow pollution and stock pollution. They link between the current damage and the future damage of stock pollution. However, they have not analyzed how the coexistence of flow pollution and stock pollution influences the contemporary trading ratios. Their design is also complicated in management of permits in the sense that permits are separated into two types: flow permit and stock permit, even though they come from the same source of emission.

This paper contributes to the current design of trading ratios by incorporating the difference between flow pollution and stock pollution, and solving the non-uniform mixing problem of water pollution. It uses a single type of permit which simplifies the design of Leiby and Rubin (2001), and focuses on changes in the contemporary trading ratios caused by the difference in the persistence of environmental damage. This paper also develops a dynamic model to include the persistence of environmental damage in the analysis of permit trading, and discusses on attaining the cost-effective outcome of pollution control via permit trading when stock pollution exists.

In this paper, a set of damage-based trading ratios is developed to attain the cost-effective outcome of pollution control. This set of trading ratios captures heterogeneity in the marginal environmental damage of pollutants from different locations, not only in the current period but also in future periods. In a simulation of permit trading, the equilibrium permit prices are compared in the scenarios where heterogeneity in the persistence of environmental damage is considered and ignored. The ignorance could distort the permit prices and generate a high efficiency loss for pollution control. The size of such efficiency loss depends on the decay rate of pollutants, the discount rate and the initial pollutant load.

The paper is organized in the following order. Section 2 establishes a cost-effective approach to pollution control from the perspective of the regulator. Section 3 develops damage-based trading ratios to solve both flow pollution and stock pollution,

[2] In fact, not all pollutants are easily carried away by water flows. For example, phosphorus can be incorporated with sediments, which are insoluble and accumulative in rivers. In this sense, riverine phosphorus is more like stock pollutant than flow pollutant. However, it does not change the essence of the following analysis in the paper.

[3] The environmental damage of SO_2 is flow pollution in the paper of Muller and Mendelsohn (2009). They have not considered acidification of soil and water bodies caused by SO_2, which is stock pollution.

[4] In fact, dynamic models are not rare in the literature on WQT. There is an increasing number of studies on dynamic permit trading since 1996, where the dynamic process results from permit banking (Hasegawa and Salant, 2015). These studies focus on the time path of pollution and permit prices, as well as decentralized behaviors of polluters (e.g. Rubin, 1996; Cronshaw and Kruse, 1996). However, this dynamic process is irrelevant to the one discussed in the paper, which comes from the continuous accumulation of the pollutant. For parsimony, this paper does not consider permit banking or borrowing.

and analyzes the equilibrium of permit trading. Section 4 runs a simulation to display how the difference between flow pollution and stock pollution impacts on the equilibrium of permit trading. Section 5 concludes.

2. Cost-effectiveness problem

Suppose that the regulator intends to control pollution in both a river and a receiving lake. This paper discusses a river-lake scenario, but "lake" can refer to any other area where pollutants end up and accumulate. Relevant polluters are indexed by $i = 1, 2, \ldots, n$ from the upstream to the downstream, and the nth polluter locates closest to the lake. The decay rate of the pollutant is γ. It tells how much the pollutant is left in one period. It is determined by physicochemical properties of the pollutant and the aquatic ecosystem. The discount rate r describes monetary depreciation in one period. Transfer coefficient τ_{ij} depicts how much the pollutant from polluter i, after decaying and being assimilated in the river, ends up arriving at polluter j. Apparently, $\tau_{ii} = 1$ and $\tau_{ij} \le 1$.[5] The transfer coefficient to the lake is denoted as τ_{is}. If the nth polluter is located at the lake, then $\tau_{ns} = 1$. The unregulated emission level of polluter i is e_i^0. The pollutant stock in the lake in period t is $S(t)$, and the initial pollutant stock is S_0. Polluter i's abatement effort is $a_i(t)$, and his abatement cost is $C_i\left(a_i(t)\right)$. Assume that all abatement costs are increasing and convex in abatement.

The environmental damage of flow pollution and the environmental damage of stock pollution are both assumed to be linear in the pollutant concentration. A non-linear function, although more precise in describing environmental damage, is also more complicated yet does not fundamentally change the analysis of trading ratios in the paper. Under non-linearity, the marginal environmental damage of polluters can be used to derive cost-effective trading ratios (Muller and Mendelsohn, 2009; Holland and Yates, 2015). However, the initial allocation of permits is much more complicated under non-linear environmental damage, as it becomes more difficult to ensure environmental damage not increase during permit trading (Konishi et al., 2015). Therefore, this paper assumes linear environmental damage for parsimony, but the analysis on trading ratios can be applied to non-linear environmental damage.

The linearity assumption on environmental damage is also common in the literature. For example, Hoel and Schneider (1997) use linear environmental cost to analyze participation and cooperation in international environmental agreement.[6] Scientific research also finds a linear relationship between pollution and the corresponding damage to health in some scenarios: Mazumder (2005) finds that the incidence of hepatomegaly (i.e. an abnormally enlarged liver) is linearly related to exposure of arsenic in drinking water; Ergene et al. (2007) discover a significant linear relationship between total heavy metal concentration and the frequencies of erythrocytic nuclear abnormalities of several fish species.

With a slight abuse of terminology, two concepts are defined here:

- River damage: the environmental damage caused by river pollution (or flow pollution);
- Lake damage: the environmental damage caused by lake pollution (or stock pollution).

Given the linearity assumption above, the marginal river damage and the marginal lake damage are constant. Denote d_i as the marginal river damage caused by polluter i, measured in dollar per unit of the pollutant. This coefficient is calculated in the way defined by Farrow et al. (2005), which integrates all the downstream river damage caused by polluter i. In period t, polluter i generates the river damage $D_i(t) = d_i(e_i^0 - a_i(t))$. The marginal lake damage is δ, also measured in dollar per unit of the pollutant. The lake damage is $\delta S(t)$ in period t.

Although the narrative of the paper describes a river-lake scenario, "lake" in the discussion can refer to a broader concept such as estuary, near shore, aquifer, or ocean. The word "lake" is adopted as a shorthand for this larger class of problems.

2.1. Flow pollution only

Start from a cost-effectiveness problem which only considers the river damage. The regulator aims to minimize the total abatement cost under the limit $\overline{D}_F$ on the river damage in each period. The problem is as below[7]:

[5] Farrow et al. (2005) describe the computation method of transfer coefficients in two steps. First, calculate the pollutant concentration C (mg/L) at distance n (m) downstream from polluter i:

$$C_{ni} = \frac{e_i}{Q} e^{-k\theta^{(T-20)} \frac{n}{U}},$$

where e_i (kg/day) is the discharge rate at polluter i, Q (m^3/day) is the stream flow, k (day^{-1}) is the nominal decay rate, θ is the sensitivity coefficient of k to a temperature T (Celsius), and U (meters/day) is the stream velocity. Second, compute the transfer coefficient between polluter i and polluter j using: $\tau_{ij} = \frac{C_{ji}}{C_{0i}}$. This transfer coefficient can be improved by including the time effect of the pollutant moving from polluter i to polluter j. The modified transfer coefficient is $\tau_{ij} e^{-rq}$, where r is the discount rate, and q is the average time for the pollutant to flow from polluter i to polluter j.

[6] To name a few more examples with the linearity assumption on environmental damage: Masoudi et al. (2015) assume the environmental damage of stock pollution can be approximated by a linear function. Specifically, Matsueda et al. (2006) and Dellink et al. (2008) adopt a linear damage function of CO_2. Ribaudo et al. (1994) impose a linear damage function of soil erosion and sediment discharge.

[7] Since it is a static optimization problem, the time variable t is eliminated in the problem.

$$\min_{\{a_1,a_2,\dots,a_n\}} \sum_{i=1}^{n} C_i(a_i)$$
$$s.t. \quad \sum_{i=1}^{n} d_i\left(e_i^0 - a_i\right) \le \overline{D}_F, \tag{A}$$
$$a_i \ge 0, \forall i \in \{1,2,\dots,n\}.$$

The marginal cost to reduce the river damage is equal everywhere in the solution to the problem above (Farrow et al., 2005). The corresponding least-cost abatement is $\{a_1^*(t), a_2^*(t), \dots, a_n^*(t)\}$, and the marginal cost ratio is:

$$\frac{C_i'(a_i^*)}{C_j'(a_j^*)} = \frac{d_i}{d_j}, \quad \forall i,j \in \{1,2,\dots,n\}. \tag{1}$$

2.2. Stock pollution only

Continue with a cost-effectiveness problem of controlling the lake damage. Regarding the continuous lake damage over time, the regulator limits the present value of the lake damage to $\overline{D}_S$. The pollutant accumulates according to the state equation: $\dot{S}(t) = -\gamma S(t) + \sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i(t)\right)$. The problem is as below:

$$\min_{\{a_1(t),a_2(t),\dots,a_n(t)\}} \int_0^{+\infty} \mathrm{e}^{-rt} \sum_{i=1}^{n} C_i(a_i(t))\,\mathrm{d}t$$
$$s.t. \quad \dot{S}(t) = -\gamma S(t) + \sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i(t)\right), \tag{B}$$
$$\int_0^{+\infty} \mathrm{e}^{-rt}\delta S(t)\,\mathrm{d}t \le \overline{D}_S,$$
$$a_i(t) \ge 0, \forall i \in \{1,2,\dots,n\}.$$

The integrand function in the objective is convex. The feasible set of the problem is closed and bounded, and the pollutant stock is bounded for all admissible pairs across time. By the Filippov-Cesari Existence Theorem, the solution to the problem above always exists. The most rapid approach to the steady state could achieve the optimality of this problem under certain conditions (e.g. Spence and Starrett, 1975). The steady-state solution is available in the appendix. The rest of this paper focuses on the steady-state solution. The least-cost marginal cost ratio in the steady state is:

$$\frac{C_i'(a_i^*(t))}{C_j'(a_j^*(t))} = \frac{\tau_{is}}{\tau_{js}}, \quad \forall i,j \in \{1,2,\dots,n\}. \tag{2}$$

2.3. Flow pollution and stock pollution

Given the analysis above, a cost-effectiveness problem is established which considers both the river damage and the lake damage. A limit $\overline{D}$ is imposed on the present value of the total damage over time. The problem is as below:

$$\min_{\{a_1(t),a_2(t),\dots,a_n(t)\}} \int_0^{+\infty} \mathrm{e}^{-rt} \sum_{i=1}^{n} C_i(a_i(t))\mathrm{d}t$$
$$s.t. \quad \dot{S}(t) = -\gamma S(t) + \sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i(t)\right),$$
$$\int_0^{+\infty} \mathrm{e}^{-rt}\left(\sum_{i=1}^{n} D_i(t) + \delta S(t)\right)\mathrm{d}t \le \overline{D}, \tag{C}$$
$$D_i(t) = d_i\left(e_i^0 - a_i(t)\right), \forall i \in \{1,2,\dots,n\},$$

In the steady state, the marginal cost to diminish the total damage is equal everywhere, and the least-cost marginal cost ratio is:

$$\frac{C_i'(a_i^*(t))}{C_j'(a_j^*(t))} = \frac{d_i + \frac{\delta}{r+\gamma}\tau_{is}}{d_j + \frac{\delta}{r+\gamma}\tau_{js}}, \quad \forall i,j \in \{1,2,\ldots,n\}. \tag{3}$$

The marginal cost ratios in equations (1)–(3) reflect the relationship of the steady-state abatement efforts among polluters in Problem (A)-(C) respectively, where Problem (A) only focuses on flow pollution, Problem (B) only focuses on stock pollution, and Problem (C) focuses on both.

3. Permit trading

Permit trading is a market-based instrument for pollution control, where polluters can trade permits with each other. The possession of permits endows polluters with the right to discharge a certain level of emission. In each period, polluters are endowed with permits $\bar{\boldsymbol{l}} = \{\bar{l}_1, \bar{l}_2, \ldots, \bar{l}_n\}$. Polluters can also generate permits by abating pollution.[8] Given that pollutant is not uniformly mixed, different polluters cannot trade their permits on a one-to-one basis. Trading ratios are hence introduced, which are akin to exchange rates. Denote the trading ratio between polluter i and polluter j as κ_{ij}. Taking $\kappa_{ij} = 2$ as an example, it means that one permit of polluter j allows polluter i to discharge twice as much of the emission as polluter j. Or, put it in the other way, one permit of polluter i grants polluter j the right to emit half as much as polluter i does. In permit trading, the trading ratios and the initial endowment of permits are exogenously determined by the regulator.

Assuming that there is no permit banking or borrowing, each polluter faces the following problem in each period:

$$\begin{aligned} &\min_{r_{ki},r_{si},r_{ji}} C_i(a_i) - p_i r_{si} + \sum_{j\neq i} p_j r_{ji} \\ &s.t. \quad \left(e_i^0 - r_{ki}\right) - \sum_{j\neq i} \kappa_{ji} r_{ji} \le 0, \\ &r_{si} + r_{ki} = \bar{l}_i + a_i, \\ &r_{ki}, r_{si}, r_{ji} \ge 0, \forall i \in \{1,2,\ldots,n\}, \end{aligned} \tag{D}$$

where r_{si} is the permits sold by polluter i, r_{ki} is the permits kept by polluter i, r_{ji} is the permits that polluter i purchases from polluter j, and p_i is the price on the permit of polluter i. These are all functions of time t, which should be precisely written as, for example, $r_{si}(t)$, but for parsimony t is eliminated.

Given a matrix of trading ratios $\boldsymbol{\kappa}$ and the initial endowment of permits $\bar{\boldsymbol{l}}$, an equilibrium of the permit market is a vector of permit prices $\boldsymbol{p}$, a vector of polluters' abatement $\boldsymbol{a}$ and a sequence of trading activities $\{r_{si}, \{r_{ji}\}_{j\neq i}\}_{i=1}^n$, where: $\boldsymbol{a}$ and $\{r_{si}, \{r_{ji}\}_{j\neq i}\}_{i=1}^n$ solve Problem (D) for every polluter; and $\sum_{i\neq j} r_{ji} \le r_{sj}$ for every polluter and the equality holds when $p_j > 0$. Since the feasible set of Problem (D) is compact, the objective function is continuous and convex in abatement and linear in trading activities, a permit-trading equilibrium always exists. The abatement vector $\boldsymbol{a}$ is unique in the equilibrium. The abatement and the marginal cost ratio in the equilibrium are:

$$\begin{aligned} a_i^{**} &= C_i'^{-1}(p_i), \quad \forall i \in \{1,2,\ldots,n\}, \\ \frac{C_i'(a_i^{**})}{C_j'(a_j^{**})} &= \kappa_{ij}, \quad \forall i,j \in \{1,2,\ldots,n\}. \end{aligned} \tag{4}$$

Permit banking and borrowing can be incorporated into the model by introducing permit bank. Polluters can save permits into the bank, and they can also use, sell or get a loan of permits from the bank. A positive number in the bank refers to possession of permits by the polluter, while a negative number means a debt of permits by a polluter. In this sense, the cost-minimizing problem of a polluter will become a dynamic optimization problem across periods when permit banking and borrowing are allowed. In contrast to the cost-minimizing Problem (D), if permit banking and borrowing are allowed, a polluter will not necessarily meet the constraint $\left(e_i^0 - r_{ki}\right) - \sum_{j\neq i}\kappa_{ji} r_{ji} \le 0$ in every period, and he could sell more permits than what he possesses in some periods. That is, r_{si} can be greater than $\bar{l}_i + a_i$, and r_{ki} can be negative in some periods.[9]

[8] In some literature, a permit is called "credit" when it is created by reducing pollution. It is tradable in permit trading, too.

[9] Once permit banking and borrowing are allowed, it alters the nature of environmental constraint to the extent that firms can emit more than the targeted level at some points in a period (Rubin, 1996). The constraint of pollution control is actually on the total environmental damage across time. Moreover, the cost-minimizing problem of a polluter with permit banking and borrowing becomes a dynamic optimization problem across time. With permit banking and borrowing, polluters may sub-optimally discharge excessive emissions in early periods. In order to settle this problem, Kling and Rubin (1997) propose not to use a one-to-one basis in trading across periods, but to discount the permits borrowed against future saving. It means that 1 permit borrowed from period t is discounted and equivalent to e^{-rt} permit in period 0.

3.1. Trading ratios

Trading ratios and the initial permit endowment are critical in a permit market. They directly determine whether the efficient pollution control is achievable through permit trading. To achieve that, the trading ratio in the steady state should equal the marginal cost ratio of Problem (A)–(C), given different pollution control targets.[10] This section is going to show that permit trading will preserve the steady state of the efficient pollution control, and the total amount of permits can be properly determined under different environmental targets.

Suppose in the steady state, for example, that there are two polluters in the permit market, and one polluter i purchases permits from the other polluter j. By this purchase, polluter i can discharge additional pollution $\Delta e_i(t)$, while polluter j must generate additional abatement $\Delta a_j(t)$. The ratio $\Delta a_j(t)/\Delta e_i(t)$ is equal to the trading ratio $\kappa_{ij}(t)$.

3.1.1. Flow pollution only

First begin with WQT which focuses only on flow pollution. The change in the river damage during permit trading is:

$$\Delta D_F(t) = d_i \Delta e_i(t) - d_j \Delta a_j(t).$$

Any set of permissible trades will lead to $\Delta D_F(t) = 0$, if $\kappa_{ij}(t)$ equals the least-cost ratio in equation (1). The contemporary incremental river damage in a permit market is offset. The cost-effective outcome of pollution control over river pollution is therefore achieved through WQT (Farrow et al., 2005). The proper trading ratios and the initial permit endowment in period t are as below:

$$\begin{aligned} &\kappa_{ij}(t) = \frac{d_i}{d_j}, \quad \forall i,j \in \{1,2,\ldots,n\}, \\ &\sum_{i=1}^{n} d_i \bar{l}_i(t) \le \overline{D}_F, \end{aligned} \tag{5}$$

where $\overline{D}_F$ is the limit on the river damage per period.

3.1.2. Stock pollution only

Continue with WQT which looks only at stock pollution. The environmental damage of stock pollution persists over time. For example, the additional emission $\Delta e_i(t)$ and abatement $\Delta a_j(t)$ influence the path of the pollutant stock over time. Denote $\Delta S(e_i(t), t)$ as the instantaneous change of the pollutant stock caused by $\Delta e_i(t)$, and similarly for $\Delta S(a_j(t), t)$. The increment in the discounted lake damage over time is:

$$\Delta D_S = \int_0^{+\infty} e^{-rt} \delta \Delta S(e_i(t), t)\, \mathrm{d}t - \int_0^{+\infty} e^{-rt} \delta \Delta S(a_j(t), t)\, \mathrm{d}t.$$

Any set of permissible trades will offset the continuous incremental damage over time if $\kappa_{ij}(t)$ is set to the marginal cost ratio in equation (2). The proper trading ratios and the initial permit endowment in the steady state should be set as below:

$$\begin{aligned} &\kappa_{ij}(t) = \frac{\tau_{is}}{\tau_{js}}, \quad \forall i,j \in \{1,2,\ldots,n\}, \\ &\sum_{i=1}^{n} \frac{\delta}{r+\gamma} \tau_{is} \bar{l}_i(t) \le \left(\overline{D}_S - \frac{\delta S_0}{r+\gamma} \right) r, \end{aligned} \tag{6}$$

where S_0 is the pollutant stock in period 0, and $\overline{D}_S$ is the limit on the discounted lake damage over time.

3.1.3. Flow pollution and stock pollution

For WQT which targets both flow pollution and stock pollution, the change in the total discounted damage is:

$$\Delta D = d_i \Delta e_i(t) + \int_0^{+\infty} e^{-rt} \delta \Delta S(e_i(t), t)\, \mathrm{d}t - d_j \Delta a_j(t) - \int_0^{+\infty} e^{-rt} \delta \Delta S(a_j(t), t)\, \mathrm{d}t.$$

[10] If the most rapid approach is the optimal solution to pollution control, this solution could be achieved by manually deducting a large amount of the pollutant in the lake and the river to reach the steady-state level and setting up permit trading afterwards. Or it may also be realized by modifying the trading ratios and the initial permit allocation period by period to quickly reach the steady state.

Any permissible set of trades will keep $\Delta D = 0$ if $\kappa_{ij}(t)$ equals the marginal cost ratio in equation (3). The following trading ratios and the initial permit endowment in the steady state are as below:

$$\kappa_{ij}(t) = \frac{d_i + \frac{\delta}{r+\gamma}\tau_{is}}{d_j + \frac{\delta}{r+\gamma}\tau_{js}}, \quad \forall i,j \in \{1,2,\ldots,n\},$$
$$\sum_{i=1}^{n}\left(d_i + \frac{\delta}{r+\gamma}\tau_{is}\right)\overline{l}_i(t) \le \left(\overline{D} - \frac{\delta S_0}{r+\gamma}\right)r, \tag{7}$$

where $\overline{D}$ is the limit on the total discounted damage. Based on the analysis above, there is:

Conclusion 1. *Suppose a river flows into a lake, and a pollutant causes damage both in the river and the lake. The river damage and the lake damage are both linear in concentration. Water quality trading is able to achieve the cost-effective outcome of pollution control. Regarding different aims of the regulator, that is, whether to protect the river, the lake or both, the proper trading ratios and the proper initial allocation of permits in the steady state are specified in equation (5), (6), and (7) respectively.*

3.2. Relation between flow pollution and stock pollution

The environmental damage caused by flow pollution and stock pollution does not necessarily have the same degree of severity. Stock pollution is significantly influenced by the initial pollutant stock and the decay rate of the pollutant, while flow pollution depends merely on the pollutant flow. A certain level of reduction in the emission may be sufficient to meet a target for river pollution, but not necessarily meet a target for lake pollution if the initial pollutant stock is high. When considering a broader water system, heterogeneity in the environmental damage might be even larger.

In a combined river-lake system, the regulator is inclined to set multiple aims on the river damage and the lake damage. Take the Mississippi River and the Gulf of Mexico as an example. The Mississippi River carries millions of tons of nutrients into the Gulf of Mexico every year, causing the "dead zone" around the Gulf of Mexico. A target of 45% reduction in riverine nitrogen and riverine phosphorus has been established to solve this stock pollution problem of the Gulf, and a Gulf Hypoxia Action Plan has been implemented covering all watersheds upstream to meet this target, too. In the meantime, the State governments in the Upper Mississippi River watershed, for example, Minnesota, Iowa and Illinois, have additional targets to control pollution in their local rivers. In this sense, the regulator still focuses on both flow pollution and stock pollution, but has to meet multiple constraints on the corresponding environmental damage. A simplest problem of the regulator is about separate limits on flow pollution and stock pollution, respectively. The problem is as below:

$$\min_{\{a_1(t),a_2(t),\ldots,a_n(t)\}} \int_0^{+\infty} \mathrm{e}^{-rt}\sum_{i=1}^{n} C_i(a_i(t))\mathrm{d}t$$
$$s.t. \quad \dot{S}(t) = -\gamma S(t) + \sum_{i=1}^{n}\tau_{is}\left(e_i^0 - a_i(t)\right),$$
$$\sum_{i=1}^{n} d_i\left(e_i^0 - a_i(t)\right) \le \overline{D}_F, \quad \int_0^{+\infty}\mathrm{e}^{-rt}\delta S(t)\mathrm{d}t \le \overline{D}_S, \tag{E}$$
$$a_i(t) \ge 0, \forall i \in \{1,2,\ldots,n\}.$$

where $\overline{D}_F$ is the limit on the river damage in each period, and the $\overline{D}_S$ is the limit on the discounted lake damage over time.

Solving the problem above, the marginal cost ratio in the steady state is:

$$\frac{C_i'(a_i^*(t))}{C_j'(a_j^*(t))} = \frac{\overline{\lambda}_F d_i + \frac{\overline{\lambda}_S\delta}{r+\gamma}\tau_{is}}{\overline{\lambda}_F d_j + \frac{\overline{\lambda}_S\delta}{r+\gamma}\tau_{js}}, \quad \forall i,j \in \{1,2,\ldots,n\}, \tag{8}$$

where $\overline{\lambda}_F$ is the Lagrange multiplier of the river damage constraint and $\overline{\lambda}_S$ is the Lagrange multiplier of the lake damage constraint. If one constraint becomes slack, the problem will degenerate to one of the problems discussed in Section 2. Compared with the steady-state marginal cost ratio under a single constraint on the environmental damage, the ratio in equation (8) contains additional parameters $\overline{\lambda}_F$ and $\overline{\lambda}_S$. To figure out $\overline{\lambda}_F$ and $\overline{\lambda}_S$, the abatement cost information of polluters is needed. Given the analysis above, there is:

Conclusion 2. *Suppose a river flows into a lake, and a pollutant causes damage both in the river and the lake. The river damage and the lake damage are both linear. Water quality trading is able to achieve the cost-effective outcome of pollution control under multiple constraints on the river damage and the lake damage in theory. However, it will require complete information on polluters' abatement costs so as to figure out the proper trading ratios as specified in* equation (8), *and the initial allocation of permits* $\sum_{i=1}^{n}\left(\overline{\lambda}_F d_i + \frac{\overline{\lambda}_S\delta}{r+\gamma}\tau_{is}\right)\overline{l}_i(t) \le \left(\overline{D} - \frac{\overline{\lambda}_S\delta S_0}{r+\gamma}\right)r$*, where* $\overline{D} = \overline{\lambda}_F\overline{D}_F + \overline{\lambda}_S\overline{D}_S$.

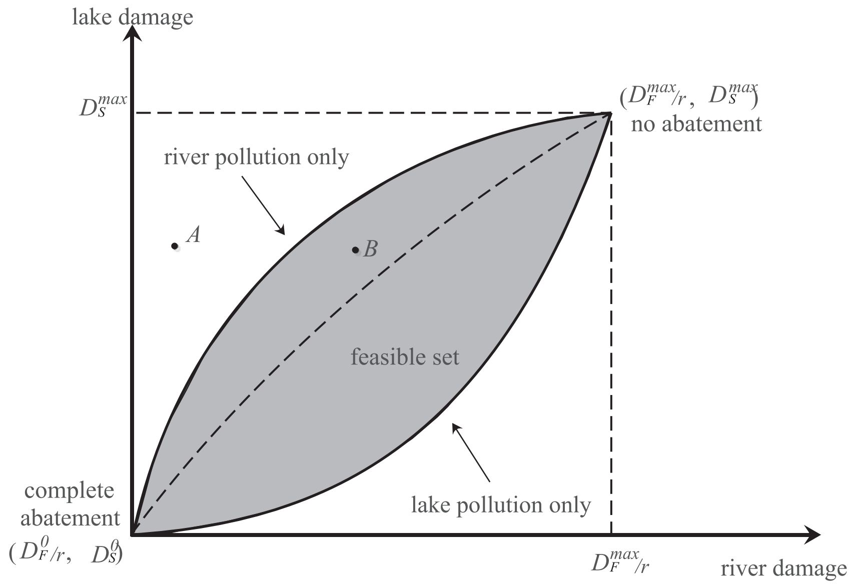

Fig. 1. Relation between river damage and lake damage.

Conclusion 2 shows the theoretical possibility of achieving the cost-effective outcome of pollution control under multiple constraints on the environmental damage. However, it also points out the practical difficulty to achieve this outcome. Although Montero (2008) develop an auction which could help collect abatement cost information of polluters at the stage of permit distribution, Conclusion 2 is more about discouraging multiple constraints on the environmental damage whenever possible.

In this paper, since the river and the lake are linked, a change in riverine water quality is directly related to a change in lake water quality. Fig. 1 shows all possible combinations of the river damage and the lake damage, which are the present values over time and are indicated by the shadow area. The lower and the upper bounds of the present value of lake damage are denoted as D_S^0 and $D_S^{\max}$ respectively. Similarly, D_F^0/r and $D_F^{\max}/r$ are the lower and the upper bounds of the present value of river damage. If only river pollution is targeted in permit trading, the efficient pollution control will move along the upper solid curve. If only lake pollution is targeted in permit trading, the efficient pollution control will move along the lower solid curve. Moreover, if the pollution targets $(\overline{D}_F/r, \overline{D}_S)$ are set outside the shadow area, for example, at the point A, the limit on lake damage will become redundant. If the targets $(\overline{D}_F/r, \overline{D}_S)$ are inside the shadow area, for example, at the point B, complete information on abatement cost will be needed to derive the efficient permit trading. If B sits closer to the upper solid curve, $\overline{D}_F$ will become more stringent to achieve than $\overline{D}_S$, and the ratio of $\overline{\lambda}_F/\overline{\lambda}_S$ will be higher. The corresponding steady-state marginal cost ratio and the total abatement cost will approach to the ones which only focus on river pollution. However, if there is a single target for the total damage of lake pollution and river pollution, the abatement cost information is not necessary to derive the efficient permit trading, which is along the dashed curve in the figure.

The analysis in this subsection calls attention to the challenges involved in setting targets in pollution control. As the scale of WQT increases, there may be more opportunities for cost saving but also more local constraints on water bodies. In order to achieve the efficient pollution control over a large area requires cooperation that coordinates individual efforts and individual targets of single areas, States, and regions. If the regulator has complete information on abatement costs of participants in permit trading, all separate targets of pollution control can be met cost-effectively. However, this is unlikely to be true in reality. In this sense, all separate targets need to be coordinated as an overall target of the whole area in order to attain the efficient pollution control. Moreover, if it is impossible to coordinate multiple targets for flow pollution and stock pollution, the analysis above also implies on trading ratios to achieve a result approaching the efficient outcome, even though these targets cannot be attained efficiently without complete information. If a target for flow pollution is more restrictive than that for stock pollution, trading ratios could be set to those with flow pollution only, i.e. equation (5). If a target for stock pollution is more restrictive than that for flow pollution, trading ratios could be set to those with stock pollution only, i.e. equation (6). If it is uncertain which target is more restrictive, trading ratios could be set to those with both flow pollution and stock pollution, i.e. equation (7).

3.3. *A little more on stock pollution*

The distinction between flow pollution and stock pollution lies in the persistence of their environmental damage, and should be considered in the non-uniform mixing problem of water pollution. This non-uniform mixing problem occurs due to a large degree of diversity in the marginal environmental damage caused by emissions from different locations. Differences in the affected areas, the physicochemical properties of pollutants and the duration of environmental damage all contribute to this diversity. Ignoring the difference between flow pollution and stock pollution, the non-uniform mixing problem could not be solved.

Besides lake pollution, stock pollution appears in many other forms in the real world. An example is arsenic contamination in groundwater. An aquifer can be regarded as a reservoir where pollutants like arsenic penetrate and accumulate. Once arsenic enters an aquifer, its effect on drinking water quality and soil quality is persistent. However, people have not paid as much attention as they should have to different types of pollution and their corresponding treatment cost. New England Interstate Water Pollution Control Commission (2001) admits that in a cooperation with the Environmental Protection Agency (EPA) to educate K-12 students in water protection, "(f)or too long, our ground water resources have been out of sight and out of mind, as is often the case, our wake-up call has come in the form of accumulated ground water pollution crises."

Indeed, considering diverse forms of stock pollution, either the existence of stock pollution or the persistence of its environmental damage are likely to be ignored sometimes in practice. Some WQT programs may consider only transfer coefficients and uncertainties between sellers and buyers when determining trading ratios, without regard to the discount rate and the initial pollutant stock. Some WQT programs may simply apply uniform trading ratios, neglecting heterogeneity of polluters (Ohio Environmental Protection Agency, 2007; Minnesota Pollution Control Agency, 2011; Corrales et al., 2013; Keller et al., 2014).

The following discusses a scenario where the regulator controls both river damage and lake damage, but fails to account for their difference in the persistence of environmental damage. The "cost-effectiveness problem" is as below:

$$\begin{aligned}
&\min_{\{a_1(t),a_2(t),\dots,a_n(t)\}} \sum_{i=1}^{n} C_i(a_i(t)) \\
&s.t. \quad \sum_{i=1}^{n} d_i\left(e_i^0 - a_i(t)\right) + \delta S(t) \le \overline{D}, \\
&S = \sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i(t)\right), \\
&a_i(t) \ge 0, \forall i \in \{1,2,\dots,n\}.
\end{aligned} \tag{F}$$

The corresponding "cost-effective" trading ratios equal the marginal cost ratios of Problem (F), which is:

$$\kappa_{ij}(t) = \frac{d_i + \delta\tau_{is}}{d_j + \delta\tau_{js}}, \forall i,j \in \{1,2,\dots,n\}. \tag{9}$$

Compared with the cost-effective trading ratios in equation (3), the decay rate γ and the discount rate r are missing in the ratios of equation (9). It is because the dynamic process and the continuous environmental damage of stock pollution are not included, when establishing permit trading. Unless $r + \gamma = 1$ or $\tau_{is} = 0, \forall i \in \{1,2,\dots,n\}$, can this trading ratio achieve the cost-effective outcome of pollution control. This gives an exception when the difference between stock pollution and flow pollution can be safely eliminated. If $r + \gamma = 1$, the continuous environmental damage of stock pollution will completely vanish after discounting and decaying. For example, $\gamma = 0$ and $r = 1$. It means that the pollutant lasts forever in the lake, but people are extremely impatient, so the present value of lake damage actually equals its damage in the current period. The influence of stock pollution is essentially instantaneous for extremely impatient people. Moreover, $\tau_{is} = 0, \forall i \in \{1,2,\dots,n\}$ means that polluters are so far upstream along the river that the pollutant arriving at the lake is trivial. Given the analysis above, there is:

Conclusion 3. *Suppose a river flows into a lake, and a pollutant causes damage both in the river and the lake. The river damage and the lake damage are both linear. If the regulator solves both flow pollution and stock pollution, but fails to account for the persistence of the environmental damage in the lake, water quality trading could not arrive at the cost-effective outcome of pollution control, unless* $r + \gamma = 1$ *or* $\tau_{is} = 0, \forall i \in \{1,2,\dots,n\}$.

4. Simulation

This section runs a simulation of permit trading on the total sewer overflow in the Upper Ohio River Basin, based on the empirical study of Farrow et al. (2005). All polluters are located along the river, whose emissions can be continuously monitored. The following section will clarify assumptions and parameters in the simulation, and will analyze the results of permit trading under different scenarios.

A homogeneous abatement cost function is assumed across polluters. A heterogenous abatement cost function is not necessary for permit trading to occur. The form of this cost function is:

$$C(a_i) = \alpha + \beta a_i + \xi\left(e_i^0 - a_i\right)\ln\left(1 - \frac{a_i}{e_i^0}\right),$$

Table 1
Parameters in the simulation on permit trading.

Parameter	Value	Parameter	Value	Parameter	Value
d_1	1.62 ($/kg)	τ_{1s}	0.10	δ	1 ($/kg)
d_2	2.21 ($/kg)	τ_{2s}	0.15	r	0.02
d_3	1.00 ($/kg)	τ_{3s}	0.20	γ	0.10
d_4	0.38 ($/kg)	τ_{4s}	0.25	e^0	231 (kg)
d_5	3.99 ($/kg)	τ_{5s}	0.30	S_0	10 (ton)
d_6	0.36 ($/kg)	τ_{6s}	0.35	α	100
d_7	1.40 ($/kg)	τ_{7s}	0.40	β	1
d_8	0.03 ($/kg)	τ_{8s}	0.45	ξ	1

where e_i^0 is the unregulated emission level, α is the fixed cost, $\alpha \geq 0$, $\beta > 0$, $\xi > 0$, and $\beta \geq \xi$. The marginal cost is non-positive and infinitely large for complete abatement.

Although the environmental damage coefficients are directly given here, it is still worthwhile to discuss how to calculate them to assist understanding. The river damage coefficient d_i integrates all the marginal environmental damage over the downstream area of the river. This coefficient is influenced by the size of the downstream area, the transfer coefficients of pollutants, and the impact of pollutants on public health and the economy. Farrow et al. (2005) compute d_i as follows:

$$d_i = \sum_{k=0}^{m} \omega \cdot h_k \cdot \tau_{ik} \cdot \frac{1}{Q_{0i}},$$

where m is the total amount on stream segments downstream of polluter i, ω ($\$ \cdot (\mathrm{kg/m^3})^{-1}$) is the willingness-to-pay of a resident for a marginal change in riverine water quality, h_k is the number of residents being affected by river pollution in the kth stream segment, Q_{0i}($\mathrm{m^3/day}$) is the stream flow of polluter i, τ_{ik} is the transfer coefficient from polluter i to the kth stream segment. Polluter i is located at the 0th stream segment. Similarly, the lake damage coefficient δ can be derived as below[11]:

$$\delta = \sum_{k=1}^{m} \omega \cdot h_k \cdot \frac{1}{Q_s},$$

where m is the total number of the segments around the lake, ω ($\$ \cdot (\mathrm{kg/m^3})^{-1}$) is the willingness-to-pay of a resident for a marginal change in lake water quality, h_k is the number of residents being affected by lake pollution in the kth segment, and Q_s($\mathrm{m^3}$) is the amount of lake water.

The parameters in the simulation are in Table 1. The river damage coefficients on the total sewer overflow in the Upper Ohio River Basin are obtained from the study of Farrow et al. (2005) (with the coefficient on Clairton, Pennsylvania being normalized to one). Further, a "lake" is assumed to locate at the end of the Ohio River. The unregulated emissions are assumed uniform among all the polluters. Other parameters, including the transfer coefficients to the lake and the lake damage coefficient, are assumed accordingly.

Given that the abatement cost functions are uniform everywhere, polluters' abatement in the efficient pollution control primarily depends on their environmental damage coefficients. Without a regulation, the river damage is $2538 per period ($126,900 in present value). The present value of lake damage across periods is $147,540. In the presence of a regulation, for example, a 30% reduction in the river damage, polluter 5 ought to abate the most to achieve the efficient pollution control over all, i.e. 106.5 kg per period. It is because given the uniform abatement cost function, the marginal river damage caused by polluter 5 is the highest among all the polluters. If a 30% reduction in the lake damage needs to be met, polluter 8 should reduce the most in a cost-effective solution, i.e. 124.9 kg per period. It is because the largest proportion of polluter 8's emission ends up in the lake. Under the linearity assumption of environmental damage, the relative marginal contribution of polluters is constant in the efficient pollution control, as it is determined by the marginal environmental damage caused by the polluters.

In the simulation, the permit prices reflect the importance of polluters in alleviating the environmental damage. Because the relative marginal contribution of polluters is constant in the efficient pollution control, the corresponding relative permit prices which reflect such marginal contribution are constant, too. Fig. 2 displays the relative permit prices to achieve the efficient pollution control, with polluter 1 being normalized to one. In a permit market which aims at river damage (river market), polluter 5 and 8 have the highest and the lowest permit prices, respectively. It is consistent with their levels of marginal river damage. Similarly, in a permit market which targets lake damage (lake market), polluter 1 and 8 have the lowest and the highest

[11] According to the definition of the lake damage coefficient, there is:

$$\delta = \sum_{k=1}^{m} \left| \frac{\partial V(W, h_k)}{\partial S} \right| = \sum_{k=1}^{m} \left| \frac{\partial V(W, h_k)}{\partial W} \frac{\partial W}{\partial C_k} \frac{\partial C_k}{\partial S} \right| = \sum_{k=1}^{m} \left| (\omega \cdot h_k) \cdot (-1) \cdot \frac{1}{Q_{ks}} \right|,$$

where $V(\cdot)$ is the benefit function of lake water quality. This benefit function depends on the lake water quality W and the amount of residents. The derivative of water quality with respect to the pollutant concentration C_k is assumed negative one uniformly.

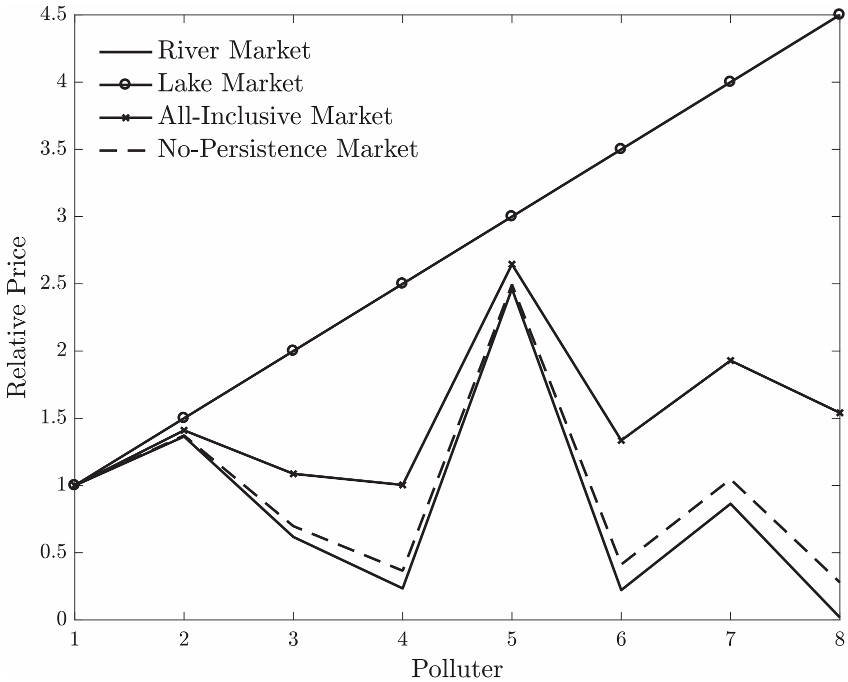

Fig. 2. Relative permit prices under different scenarios.

permit prices, resulting form their marginal lake damage. These relative permit prices correspond to the trading ratios, which are linked to the river damage coefficients and the transfer coefficients to the lake.

In a permit market which aims at both river damage and lake damage (all-inclusive market), the relative permit prices lie between those of the river market and the lake market. The relative prices take account of polluters' marginal effect on both river pollution and lake pollution.

The dashed line in Fig. 2 represents the relative prices in a permit market, where the persistence of lake damage fails to be counted (no-persistence market). Without considering the continuous effect of lake damage, polluters' ability to improve the environment is incorrectly estimated. The corresponding permit prices are distorted and unable to reflect polluters' actual influence on the environment. For example, in the all-inclusive market, polluter 1 and 8 have very similar prices, since their marginal impact on the total damage is similar. However, in the no-persistence market, polluter 8's contribution to reducing the continuous damage of lake pollution is neglected, and his permit price therefore becomes lower and closer to that in the river market.

The non-uniform mixing problem fails to be solved in the no-persistence market, and the cost-effective outcome of pollution control is not achieved. The efficiency loss is influenced by the lake damage coefficient δ, the discount rate r, the decay rate γ, and the initial pollutant stock S_0. These factors determine the continuous environmental damage of lake pollution. Fig. 3 compares the present value of total abatement cost in the no-persistence market and the all-inclusive market with respect to these four factors. The cost difference in Fig. 3 is represented as a percentage of the total abatement cost in the all-inclusive market.

The cost difference is always positive and is of an inverse-U shape. When a reduction target in the total damage (i.e. both river damage and lake damage) approaches to zero, the cost difference will go down. It is because the pollution control problem essentially degenerates to the unregulated scenario, and the polluters find it optimal not to abate; When a reduction target is sufficiently high, the cost difference will also decline since almost everyone is driven to complete abatement. Moreover, the lake damage coefficient δ, the decay rate γ and the discount rate r all can alter the maximum of the cost difference. For example, the maximum of the cost difference is higher under a lower decay rate. It is because a lower decay rate leads to a greater continuous environmental damage, and the ignorance of such continuous damage creates a larger distortion in the trading ratios from the efficient ones in the all-inclusive market and a greater efficiency loss. However, the initial pollutant stock S_0 does not change the maximum of the cost difference, in that it does not directly affect the trading ratios in the permit market. Additionally, Fig. 3 indicates that the maximum achievable reduction in pollution varies, under different values of the parameters which are related to the continuous environmental damage. For example, regarding the initial pollutant stock S_0 can be removed only through pollutant's decaying (manual removal of the pollutant in the lake is not considered here). A higher S_0 leads to a higher continuous lake damage and thus a lower level of the attainable maximum reduction for pollution control.

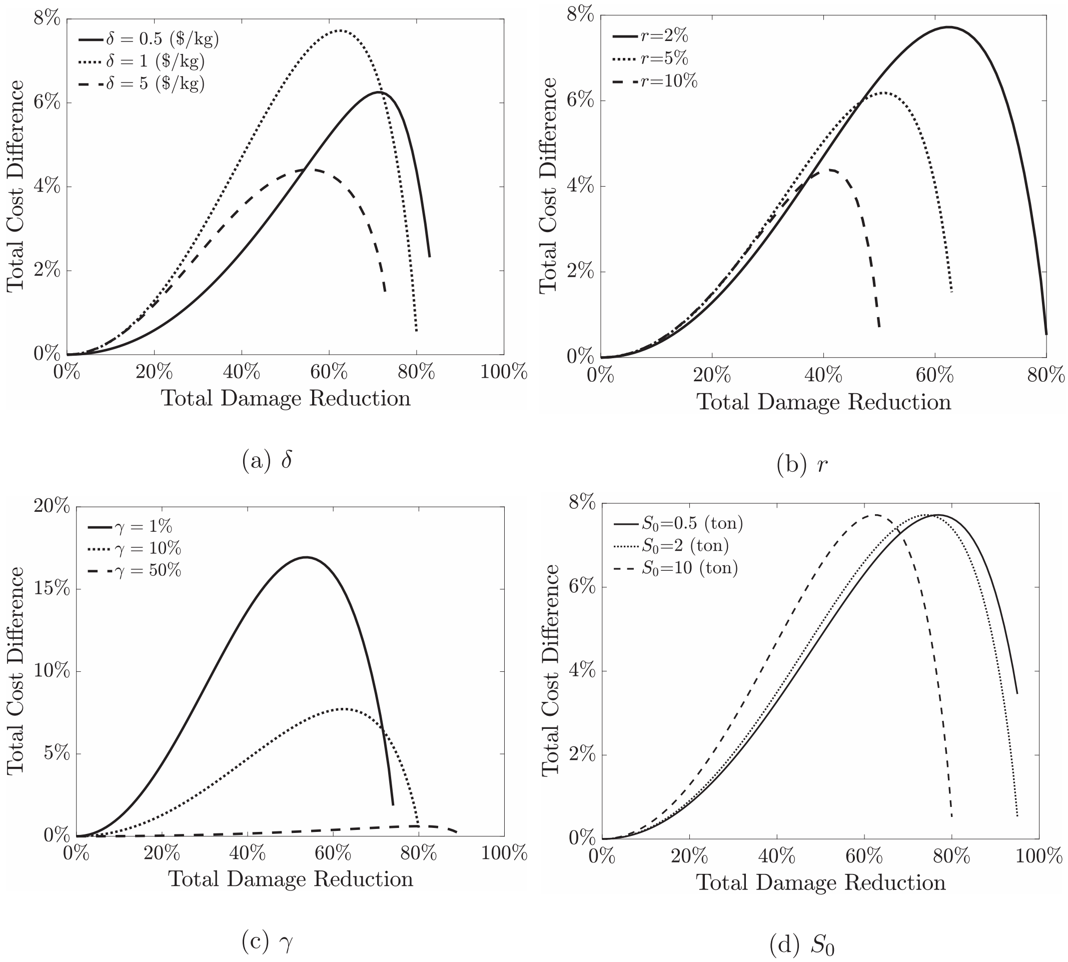

Fig. 3. The cost difference between No-persistence market and all-inclusive market.

5. Conclusion

This paper points out the importance of incorporating heterogeneity in the persistence of environmental damage in solving the non-uniform mixing problem in permit trading. It contributes to the previous literature by developing trading ratios that account for different impact of polluters on the continuous environmental damage, and ensure that the efficient pollution control is achievable through permit trading. However, this paper discourages to achieve multiple targets at flow pollution and stock pollution by permit trading, because it will require complete information on polluters' abatement costs to achieve the cost-effective outcome of pollution control.

The permit trading scheme is promising in pollution control as it may reduce transaction costs, and this paper improves permit trading by taking account of the dynamics of environmental damage. First, this improved permit trading does not require much information or workload from polluters and administrators in trades. Participants can trade their permits freely according to a predetermined matrix of trading ratios. The trading rule is easy to follow and trades are easy to process. Both help alleviate a thin market problem and decrease transaction cost (Hung and Shaw, 2005). Second, the damage-based trading-ratio system in the paper helps reduce the cost of pollution control, as the heterogeneity in the marginal environmental damage is being considered in pollution abatement. Muller and Mendelsohn (2009) show that additional cost-saving achieved in the SO_2 allowance trading program by using the damage-based trading-ratio system. Third, the difference between flow pollution and stock pollution has been incorporated into the permit trading, which was missing before. This could potentially create more opportunities for trading to solve a thin market problem. Besides, the dynamics of environmental damage has been modelled. It enables the regulator to consider the influence of permit trading on environmental damage in future.

This paper also provides opportunities for further study. The analysis in the paper is based on the linearity assumption on environmental damage. Although the design of trading ratios could be extended easily to the scenario of non-linear environmental damage, the initial allocation of permits becomes more complicated. It will be worthwhile to analyze the permit trading under a less stringent assumption on environmental damage. Moreover, this paper does not discuss stochastic factors in the environment such as weather and uncertainties in nonpoint pollution. A further improvement on trading ratios by incorporating uncertainties into the design of trading ratios will be interesting.

Appendix

Cost-Effectiveness Problem of Stock Pollution Only

In the cost-effectiveness problem of stock pollution only, the Hamiltonian function H, the first order conditions and the transversality condition are given below, where $\overline{\lambda}$ is a Lagrange multiplier:

$$H = \sum_{i=1}^{n} C_i(a_i) + \overline{\lambda}\left(\delta S - \frac{\overline{D}_S}{r}\right) + \mu\left(-\gamma S + \sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i\right)\right),$$

$$C_i'(a_i) - \mu\tau_{is} = 0, \quad \forall i \in \{1, 2, \ldots, n\},$$

$$\dot{\mu} - r\mu = \mu\gamma - \overline{\lambda}\delta,$$

$$\dot{S} = -\gamma S + \sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i\right),$$

$$\lim_{t\to\infty} e^{-rt} S(t)\mu(t) = 0.$$

Solving the differential equations, there is $\mu = \overline{\lambda}\delta / (r + \gamma)$, which is constant over time. The size of $\overline{\lambda}$ depends on $\overline{D}_S$. The constraint on stock pollution is always binding unless $\overline{D}_S$ is not stringent enough.

The Kuhn-Tucker Conditions of Problem (D)

Every polluter attempts to minimize his abatement cost and the cost incurred under permit trading. The Lagrange function of Problem (D) is:

$$L_i = C_i(a_i) - p_i r_{si} + \sum_{j\neq i} p_j r_{ji} + \lambda_i\left[\left(e_i^0 - r_{ki}\right) - \sum_{j\neq i} t_{ji} r_{ji}\right].$$

The Kuhn-Tucker Conditions are:

$$r_{ki} : C_i'(a_i) - \lambda_i \geq 0, \quad r_{ki} \geq 0, \quad \left(C_i'(a_i) - \lambda_i\right) r_{ki} = 0,$$

$$r_{si} : C_i'(a_i) - p_i \geq 0, \quad r_{si} \geq 0, \quad \left(C_i'(a_i) - p_i\right) r_{si} = 0,$$

$$r_{ji} : p_j - \lambda_i t_{ji} \geq 0, \quad r_{ji} \geq 0, \quad \left(p_j - \lambda_i t_{ji}\right) r_{ji} = 0, \forall j \neq i,$$

$$\lambda_i : \left(e_i^0 - r_{ki}\right) - \sum_j t_{ji} r_{ji} \leq 0, \lambda_i \geq 0, \left(\left(e_i^0 - r_{ki}\right) - \sum_j t_{ji} r_{ji}\right)\lambda_i = 0.$$

If the constraint λ_i is not binding, i.e. $\left(e_i^0 - r_{ki}\right) - \sum_j t_{ji} r_{ji} < 0$, λ_i will be zero. However, $\lambda_i = 0$ indicates either $p_j = 0$ or $r_{ji} = 0$ for any $j \neq i$. No market exists in this scenario. If the constraint λ_i is binding, i.e. $\left(e_i^0 - r_{ki}\right) - \sum_j t_{ji} r_{ji} = 0$, λ_i could be positive, $C_i'(a_i) - \lambda_i = 0, C_i'(a_i) - p_i = 0$, and $p_j - \lambda_i t_{ji} = 0$ due to non-arbitrage condition.[12]

The Initial Allocation of Permits in WQT, Which Focuses on Both Flow Pollution and Stock Pollution

Each polluter in a permit market must meet the requirement that he cannot discharge more than what is allowed by the permits he possesses. That is,

$$\left(e_i^0 - r_{ki}\right) - \sum_{j\neq i} t_{ji} r_{ji} \leq 0, \quad \forall i \in \{1, 2, \ldots, n\}.$$

Because $r_{si} + r_{ki} = \overline{l}_i + a_i$, the above inequality is rearranged as follows:

$$\left(e_i^0 - a_i\right) \leq \overline{l}_i - r_{si} + \sum_{j\neq i} t_{ji} r_{ji}, \quad \forall i \in \{1, 2, \ldots, n\}$$

[12] If $C_i'(a_i) - \lambda_i = 0$ and $C_i'(a_i) - p_i = 0$, then $p_j - \lambda_i t_{ji} = 0$. See the details in Konishi et al. (2015).

Multiplying both sides by the river damage coefficient d_i, aggregating over all polluters and discounting over time, there is an inequality for the environmental damage of flow pollution:

$$\int_0^{+\infty} \sum_{i=1}^{n} d_i \left(e_i^0 - a_i\right) e^{-rt} dt \le \int_0^{+\infty} \sum_{i=1}^{n} d_i \left(\bar{l}_i - r_{si} + \sum_{j \ne i} t_{ji} r_{ji}\right) e^{-rt} dt. \tag{10}$$

According to the state equation $\dot{S} = -\gamma S + \sum_{i=1}^{n} \tau_{is}(e_i^0 - a_i)$, the discounted environmental damage of stock pollution is:

$$\begin{aligned}\int_0^{+\infty} \delta S e^{-rt} dt &= \int_0^{+\infty} \delta \left[\left(S_0 - \frac{1}{\gamma}\sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i\right)\right) e^{-\gamma t} + \frac{1}{\gamma}\sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i\right)\right] e^{-rt} dt \\ &= \frac{\delta S_0}{r+\gamma} + \frac{\delta}{r(r+\gamma)} \sum_{i=1}^{n} \tau_{is}\left(e_i^0 - a_i\right) \le \frac{\delta S_0}{r+\gamma} + \frac{\delta}{r(r+\gamma)} \sum_{i=1}^{n} \tau_{is}\left(\bar{l}_i - r_{si} + \sum_{j\ne i} t_{ji} r_{ji}\right).\end{aligned} \tag{11}$$

Combining inequalities (10) and (11), there is an inequality describing the discounted environmental damage of both flow pollution and stock pollution:

$$\begin{aligned}&\int_0^{+\infty} \sum_{i=1}^{n} d_i \left(e_i^0 - a_i\right) e^{-rt} dt + \int_0^{+\infty} \delta S e^{-rt} dt \\ &\le \int_0^{+\infty} \sum_{i=1}^{n} d_i \left(\bar{l}_i - r_{si} + \sum_{j\ne i} t_{ji} r_{ji}\right) e^{-rt} dt + \frac{\delta S_0}{r+\gamma} + \frac{\delta}{r(r+\gamma)} \sum_{i=1}^{n} \tau_{is}\left(\bar{l}_i - r_{si} + \sum_{j\ne i} t_{ji} r_{ji}\right) \\ &= \frac{1}{r}\sum_{i=1}^{n}\left(d_i + \frac{\delta \tau_{is}}{r+\gamma}\right)\bar{l}_i - \frac{1}{r}\sum_{i=1}^{n}\left(d_i + \frac{\delta\tau_{is}}{r+\gamma}\right)\left(r_{si} - \sum_{j\ne i} t_{ji} r_{ji}\right) + \frac{\delta S_0}{r+\gamma}.\end{aligned}$$

In the equilibrium of permit trading, if permit prices are nonzero, there will be $r_{sj} = \sum_{i\ne j} r_{ji}$, $\forall j \in \{1, 2, \ldots, n\}$. Given the efficient trading ratio $t_{ij} = C_i'(a_i^*)/C_j'(a_j^*)$, there are:

$$\begin{aligned}&\frac{1}{r}\sum_{i=1}^{n}\left(d_i + \frac{\delta\tau_{is}}{r+\gamma}\right)\left(r_{si} - \sum_{j\ne i} t_{ji} r_{ji}\right) = \frac{1}{r}\left(\sum_{i=1}^{n}\left(d_i + \frac{\delta\tau_{is}}{r+\gamma}\right) r_{si} - \sum_{i=1}^{n}\sum_{j\ne i}\left(d_i + \frac{\delta\tau_{is}}{r+\gamma}\right) t_{ji} r_{ji}\right) \\ &= \frac{1}{r}\left(\sum_{j=1}^{n}\left(d_j + \frac{\delta\tau_{js}}{r+\gamma}\right) r_{sj} - \sum_{j=1}^{n}\sum_{i\ne j}\left(d_i + \frac{\delta\tau_{is}}{r+\gamma}\right) t_{ji} r_{ji}\right) = \frac{1}{r}\sum_{j=1}^{n}\left(d_j + \frac{\delta\tau_{js}}{r+\gamma}\right)\left(r_{sj} - \sum_{i\ne j} r_{ji}\right) = 0.\end{aligned}$$

Since $\bar{D}$ is the limit on the total damage, there is $\int_0^{+\infty} \sum_{i=1}^{n} d_i \left(e_i^0 - a_i\right) e^{-rt} dt + \int_0^{+\infty} \delta S e^{-rt} dt \le \bar{D}$. Therefore, to achieve the efficient result of pollution control with a limit $\bar{D}$, the initial allocation of permits in WQT shall be:

$$\sum_{i=1}^{n}\left(d_i + \frac{\delta}{r+\gamma}\tau_{is}\right)\bar{l}_i \le \left(\bar{D} - \frac{\delta S_0}{r+\gamma}\right) r.$$

Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.jeem.2018.07.002.

References

Corrales, J., Naja, G.M., Rivero, R.G., Miralles-Wilhelm, F., Bhat, M.G., 2013. Water quality trading programs towards solving environmental pollution problems. Irrigat. Drain. 62, 72–92.

Cronshaw, M.B., Kruse, J.B., 1996. Regulated firms in pollution permit markets with banking. J. Regul. Econ. 9 (2), 179–189.

Dechert, W., O'Donnell, S., 2006. The stochastic lake game: a numerical solution. J. Econ. Dynam. Contr. 30, 1569–1587.

Dellink, R., Finus, M., Olieman, N., 2008. The stability likelihood of an international climate agreement. Environ. Resour. Econ. 39, 357–377.

Ergene, S., Çava, T., Çelik, A., Köleli, N., Kaya, F., Karahan, A., 2007. Monitoring of nuclear abnormalities in peripheral erythrocytes of three fish species from the Goksu Delta (Turkey): genotoxic damage in relation to water pollution. Ecotoxicology 16, 385–391.

Farrow, R.S., Schultz, M.T., Celikkol, P., Houtven, G.L.V., 2005. Pollution trading in water quality limited areas: use of benefits assessment and cost-effective trading ratios. Land Econ. 81 (2), 191–205.

Fisher-Vanden, K., Olmstead, S., 2013. Moving pollution trading from air to water: potential, problems, and prognosis. J. Econ. Perspect. 27 (1), 147–172.

Hasegawa, M., Salant, S., 2015. The dynamics of pollution permits. Annu. Rev. Resour. Econ. 7 (1), 61–79.

Hediger, W., 2009. Sustainable development with stock pollution. Environ. Dev. Econ. 14 (6), 759–780.
Hoel, M., Karp, L., 2001. Taxes and Quotas for A Stock pollutant with multiplicative uncertainty. J. Publ. Econ. 82, 91–114.
Hoel, M., Schneider, K., 1997. Incentives to participate in an international environmental agreement. Environ. Resour. Econ. 9, 153–170.
Holland, S.P., Yates, A.J., 2015. Optimal trading ratios for pollution permit markets. J. Publ. Econ. 125, 16–27.
Horan, R.D., Shortle, J.S., 2011. Economic and ecological rules for water quality trading. JAWRA J. Am. Water Res. Assoc. 47 (1), 59–69.
Hung, M.F., Shaw, D., 2005. A trading-ratio system for trading water pollution discharge permits. J. Environ. Econ. Manag. 49 (1), 83–102.
Iwasa, Y., Uchida, T., Yokomizo, H., 2007. Nonlinear behavior of the socio-economic dynamics for lake eutrophication control. Ecol. Econ. 63, 219–229.
Keller, A.A., Chen, X., Fox, J., Fulda, M., Dorsey, R., Seapy, B., Glenday, J., Bray, E., 2014. Attenuation coefficients for water quality trading. Environ. Sci. Technol. 48 (12), 6788–6794.
Kling, C., Rubin, J., 1997. Bankable permits for the control of environmental pollution. J. Publ. Econ. 64 (1), 101–115.
Konishi, Y., Coggins, J., Wang, B., 2015. Water quality trading: can we get the prices of pollution right? Water Resour. Res. 1–19.
Laukkanen, M., Huhtala, A., 2008. Optimal Management of A Eutrophied coastal ecosystem: balancing agricultural and municipal abatement measures. Environ. Resour. Econ. 39, 139–159.
Leiby, P., Rubin, J., 2001. Intertemporal permit trading for the control of greenhouse gas emissions. Environ. Resour. Econ. 19 (3), 229–256.
Lieb, C.M., 2004. The environmental kuznets curve and flow versus stock pollution: the neglect of future damages. Environ. Resour. Econ. 29 (4), 483–506.
Masoudi, N., Santugini, M., Zaccour, G., 2015. A dynamic game of emissions pollution with uncertainty and learning. Environ. Resour. Econ. 1–24.
Matsueda, N., Futagami, K., Shibata, A., 2006. Environmental transfers against global warming: a credit-based program. Int. J. Global Environ. Issues 6, 47–72.
Mazumder, D.G., 2005. Effect of chronic intake of arsenic-contaminated water on liver. Toxicol. Appl. Pharmacol. 206, 169–175.
Mesbah, S.M., Kerachian, R., Nikoo, M.R., 2009. Developing real time operating rules for trading discharge permits in rivers: application of Bayesian Networks. Environ. Model. Software 24 (2), 238–246.
Minnesota Pollution Control Agency, 2011. Proposed Permanent Rules Relating to Water Quality Trading 7054. http://www.pca.state.mn.us/index.php/view-document.html?gid=15816.
Montero, J.P., 2008. A simple auction mechanism for the optimal allocation of the commons. Am. Econ. Rev. 98 (1), 496–518.
Montgomery, W.D., 1972. Market in licenses and efficient pollution control programs. J. Econ. Theor. 8, 395–418.
Muller, N.Z., Mendelsohn, R., 2009. Efficient pollution regulation: getting the prices right. Am. Econ. Rev. 99 (5), 1714–1739.
New England Interstate Water Pollution Control Commission, 2001. Ground water contamination: getting up to speed. In: The Magnificent Ground Water Connection, pp. 243–250. https://www.epa.gov/education/magnificent-ground-water-connection.
Nguyen, N., Shortle, J., Reed, P., Nguyen, T., 2013. Water quality trading with asymmetric information, uncertainty and transaction costs: a stochastic agent-based simulation. Resour. Energy Econ. 35 (1), 60–90.
Obropta, C.C., Rusciano, G.M., 2006. Addressing total phosphorus impairments with water quality trading. J. Am. Water Resour. Assoc. 42 (5), 1297–1306.
Ohio Environmental Protection Agency, 2007. The Ohio Administrative Code Chapter 3745-5 Water Quality Trading. http://www.epa.state.oh.us/dsw/rules/3745_5.aspx.
Ribaudo, M.O., Osborn, C.T., Konyar, K., 1994. Land retirement as a tool for reducing agricultural nonpoint-source pollution. Land Econ. 70 (1), 77–87.
Roberts, D., Clark, C., 2008. A spatial assessment of possible water quality trading markets in Tennessee. Rev. Agric. Econ. 30 (4), 711–728.
Rubin, J.D., 1996. A model of intertemporal emission trading, banking, and borrowing. J. Environ. Econ. Manag. 31 (3), 269–286.
Schrauben, M., 2010. Pollution in the Grand River. http://therapidian.org/water-quality-grand-river. [Accessed 9 October 2014].
Spence, M., Starrett, D., 1975. Most rapid approach paths in accumulation problems. Int. Econ. Rev. 16 (2), 388–403.