Example: When an urban area begins to encroach on the boundary of an old growth forest, the government faces the decision of continuing preservation versus clearing the forest for development.

Use a 2-period model

- Area of land =1
- Two alternative uses: Preservation (current use) and Development.
- D_t = 1 if development in period t, = 0 otherwise.
- d_t: benefit of development in period t, t = 1, 2.
- w_t: wilderness benefit in period t such as recreational benefits and (the probability of) finding pharmaceuticals such as cure-for-cancer.
- c_t: cost of investment for development.
- All benefits/costs are in present value terms. Discount factor already included.
- Assumptions

❖ <u>Irreversibility</u>: Development may be technically reversible. Cost of un-paving and converting back to forest is high. In addition, the length of time required for regeneration of the forest is so great that, given some positive rate of time preference, it might as well be irreversible.

❖ <u>Uncertainty</u>: w_2 is unknown in period 1. w_2 is the opportunity cost of development.

❖ <u>Information structure 1</u>: uncertainty about w_2 is resolved at the beginning of the 2nd period. So w_2 will be known when decision needs to be made (and acquisition of information is independent of choices of development or preservation, no learning by doing).

❖ <u>Information structure 2</u>: uncertainty about w_2 remains in period 2.

	Scenario 1	Scenario 2	Scenario 3
	V_1^*	V_1^c	V_1^{NPV}
Irreversibility	Development irreversible	Development irreversible	Development irreversible
Uncertainty [Learning]	Information structure 1	Information structure 2	Information structure 1
Flexibility	Develop Now or Later	Develop Now or Later	Develop Now or Never

Notations:

o $V_1^*(0)$: Scenario 1 value function if $D_1 = 0$. $V_1^*(0) = w_1 + E_1[\text{Max}(d_2 - c_2, w_2)]$

o $V_1^c(0)$: Scenario 2 value function if $D_1 = 0$. $V_1^c(0) = w_1 + \text{Max}(d_2 - c_2, E_1[w_2]$

o $V_1^{NPV}(0)$: Scenario 3 value function if $D_1 = 0$. $V_1^{NPV}(0) = w_1 + E_1[w_2]$

o $V(1) = V_1^*(1) = V_1^c(1) = V_1^{NPV}(1)$Numerical example: $d_1 = d_2 = 100$, $c_1 = c_2 = 20$, $w_1 = 60$, $w_2 =$

$$\begin{cases} 0 & \text{with prob } 1/2 \\ 200 & \text{with prob } 1/2 \end{cases}$$

Convert to Continuous case (Mezey and Conrad. 2010. Real Options in Resource Economics. Section 2. Forestry).

- *A*: a constant flow of amenity value.
- *N*(*t*): net benefit if clear cut the old growth forest. Once clear cut, the amenity value is lost forever.
- Geometric Brownian motion: $dN = \mu Ndt + \sigma Ndz$, where dz is the increment of a Brownian motion.
- δ: instantaneous discount rate

$$\delta V(N) = A + (1/dt)\mathcal{E}\{dV\}, \tag{5}$$

With $N = N(t)$ evolving according to Equation 1, Itô's lemma implies that

$$(1/dt)\mathcal{E}\{dV\} = \mu NV'(N) + (\sigma^2/2)N^2 V''(N), \tag{6}$$

which upon substitution into Equation 5 yields

$$(\sigma^2/2)N^2 V''(N) + \mu NV'(N) - \delta V(N) = -A. \tag{7}$$

Equation 7 is a second-order, nonhomogeneous, differential equation in the unknown value function, $V(N)$. The solution to Equation 7 is the function

$$V(N) = BN^\beta + A/\delta, \tag{8}$$

where $B > 0$ is an unknown constant and

$$\beta = (1/2 - \mu/\sigma^2) + \sqrt{(1/2 - \mu/\sigma^2)^2 + 2\delta/\sigma^2} > 1. \tag{9}$$