

In class exercise

Q1.

A. C: chicken sandwich, F: French Fries

Budget constraint: $6C + 2F \leq 18$

B. $P(C)=6, P(F)=2$, C: chicken sandwich quantity, F: French Fries quantity

U(C): Utility of chicken sandwich, U (F): Utility of French Fries

MU(C): Marginal Utility of chicken sandwich, MU (F): Marginal Utility of French Fries

MU(C)/P(C): Marginal Utility of chicken sandwich per dollar

MU (F)/P(F): Marginal Utility of French Fries per dollar

Method 1: PRICINPLE OF RATIONAL CHOICE(not recommended!). This will not give your correct answers all the time, unless you have a continuously differentiable utility function. (Don't worry if you don't know "continuously differentiable".) You need to use method 2 to check it. By the principle of rational choice,

$$MRS_{FC} = \frac{P_F}{P_C} = \frac{2}{6} = \frac{1}{3}$$

Method 2: GO THROUGH THE TABLE. This method is quite messy, but it will always give you the correct answer.

Based on the conditions, we have the following table.

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	2.5	1	11	11	5.5
2	25	10	1.67	2	21	10	5
3	31	6	1	3	30	9	4.5
4	34	3	0.5	4	37	7	3.5
5	36	2	0.33	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

In order to maximize the total utility, you need to maximize the utility you obtain from each dollar you spend, i.e. try to maximize (MU/P) of each dollar when you decide whether to buy chicken sandwich or French fries. Follow the following steps:

- a. Decide where to spend the first dollar of \$18: chicken sandwich or French fries? (See Figure 1) Because $MU(F)/P(F)=5.5 > MU(C)/P(C)=2.5$, so buy French fries first. You spend \$2 buying one unit of French fries, then you have \$ 16 (=\$18-\$2) left.

- b. Then decide where to spend your next dollar. (See Figure 2) Because $MU(F)/P(F)$ of the second unit French fries = 5 > $MU(C)/P(C) = 2.5$, so buy second unit of French fries. You spend \$2 buying one unit of French fries, then you will have \$ 14 (= \$16 - \$2) left.
- c. Repeat the same steps as above. After you bought 6th unit of French fries, you have \$6 (= \$18 - \$12) left. Then you determine where to spend your next dollar. (See Figure 3) Because $MU(F)/P(F)$ of the 7th unit French fries = 2 < $MU(C)/P(C) = 2.5$, so buy one unit of chicken sandwich. You spend \$6 in buying one unit of chicken sandwich, then you have \$0 (= \$6 - \$6)

In a word, the consumption bundle to maximize your utility is 1 unit of chicken sandwich and 6 units of French fries. The Marginal utility of 1st unit of chicken sandwich is 15, the Marginal utility of 6th unit of chicken sandwich is 5. Then,

$$MRS_{FC} = \frac{MU_F}{MU_C} = \frac{5}{15} = \frac{1}{3}$$

(We see that method 2 and method 1 have the same answer in this problem, but you will see they are different in the next problem.)

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	2.5	1	11	11	5.5
2	25	10	1.67	2	21	10	5
3	31	6	1	3	30	9	4.5
4	34	3	0.5	4	37	7	3.5
5	36	2	0.33	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Figure 1

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	2.5	1	11	11	5.5
2	25	10	1.67	2	21	10	5
3	31	6	1	3	30	9	4.5
4	34	3	0.5	4	37	7	3.5
5	36	2	0.33	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Figure 2

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	2.5	1	11	11	5.5
2	25	10	1.67	2	21	10	5
3	31	6	1	3	30	9	4.5
4	34	3	0.5	4	37	7	3.5
5	36	2	0.33	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Figure 3

C. $P(C)=3$, $P(F)=2$, C: chicken sandwich quantity, F: French Fries quantity

Method 1: (not recommended!)

Method 2:

Based on the conditions, we have the following table.

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	5	1	11	11	5.5
2	25	10	3.33	2	21	10	5
3	31	6	2	3	30	9	4.5
4	34	3	1	4	37	7	3.5
5	36	2	0.67	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Similar to above, follow the following steps:

- Decide where to spend the first dollar of \$18: chicken sandwich or French fries? (See Figure 4) Because $MU(F)/P(F)=5.5 > MU(C)/P(C)=5$, so buy French fries first. You spend \$2 buying one unit of French fries, then you will have \$ 16 ($=\$18-\2) left.
- Repeat the same steps as above. After you bought 2nd unit of French fries, you have \$14 ($=\$18-\4) left. Then decide where to spend your next dollar. (See Figure 5) Because $MU(F)/P(F)$ of 3rd unit French fries= $4.5 < MU(C)/P(C)=5$, so buy one unit of chicken sandwich. You spend \$3 buying one unit of French fries, then you have \$ 11 ($=\$14-\3) left.
- Then decide where to spend your next dollar. (See Figure 6) Because $MU(F)/P(F)$ of 3rd unit French fries= $4.5 > MU(C)/P(C)$ of 2nd unit chicken sandwich= 3.33 , so buy 3rd unit of

French fries. You spend \$2 buying one unit of French fries, then have \$ 9 (=\$11-\$2) left. By the same token, you buy 4th unit of French fries and have \$7 (=\$9-\$2) left.

- d. Then decide where to spend your next dollar. Because MU (F)/P(F) of 5th unit French fries=2.5<MU(C)/P(C) of 2nd unit chicken sandwich=3.33, so buy 2nd unit of chicken sandwich. You spend \$3 buying one unit of French fries, then you will have \$ 4 (=\$7-\$3) left.
- e. Then decide where to spend your next dollar. Because MU (F)/P(F) of 5th unit French fries=2.5>MU(C)/P(C) of 3rd unit chicken sandwich=2, so buy 5th unit of French fries. You spend \$2 buying one unit of French fries and have \$ 2 (=\$4-\$2) left. By the same token, you buy 6th unit of French fries and have \$2 (=\$2-\$2) left.

In a word, the consumption bundle to maximize your utility is 2 unit of chicken sandwich and 6 units of French fries. The Marginal utility of 2nd unit of chicken sandwich is 10, the Marginal utility of 6th unit of chicken sandwich is 5. Then,

$$MRS_{FC} = \frac{MU_F}{MU_C} = \frac{5}{10} = \frac{1}{2}$$

(Note this is different from $\frac{P_F}{P_C} = \frac{2}{3}$)

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	5	1	11	11	5.5
2	25	10	3.33	2	21	10	5
3	31	6	2	3	30	9	4.5
4	34	3	1	4	37	7	3.5
5	36	2	0.67	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Figure 4

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	5	1	11	11	5.5
2	25	10	3.33	2	21	10	5
3	31	6	2	3	30	9	4.5
4	34	3	1	4	37	7	3.5
5	36	2	0.67	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Figure 5

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	5	1	11	11	5.5
2	25	10	3.33	2	21	10	5
3	31	6	2	3	30	9	4.5
4	34	3	1	4	37	7	3.5
5	36	2	0.67	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Figure 6

Some extra exercise

This problem could be more difficult if it is given as follows: $P(C)=3$, $P(F)=2$, C: chicken sandwich quantity, F: French Fries quantity. Please find out the consumption bundle that maximizes total utility and the corresponding MRS_{FC} . (Try to do it by yourself and check the answers to see whether you master this method or not.)

C	U(C)	MU(C)	MU(C)/P(C)	F	U(F)	MU(F)	MU(F)/P(F)
0	0			0	0		
1	15	15	5	1	11	11	5.5
2	25	10	3.33	2	21	10	5
3	34	9	3	3	30	9	4.5
4	37	3	1	4	37	7	3.5
5	39	2	0.67	5	42	5	2.5
				6	47	5	2.5
				7	51	4	2
				8	53	2	1
				9	55	2	1
				10	56	1	0.5

Answers:

- Repeat the same steps of problem C until step e.
- Now you have bought 2 chicken sandwiches and 4 French fries, and \$4 dollars left. Because $MU(F)/P(F)$ of 5th unit French fries = 2.5 < $MU(C)/P(C)$ of 3rd unit chicken sandwich = 3, so buy 3rd unit of chicken sandwich. You spend \$3 buying one unit of chicken sandwich and have \$1 (= \$4 - \$3) left. You can buy nothing more and the utility of your last \$4 is 9 (which is $MU(C)$ of 3rd unit of chicken sandwich). **Since you haven't used up all you budget in this consumption bundle (let's call it Choice 1), you have another choice that clears the budget:** buy two more units (i.e. 5th and 6th) of French

fries. Then the utility of your last \$4 is 10 (=5+5, which is the sum of MU(F) from 5th and 6th unit of French fries). This utility is bigger than that of choice 1.

Hence, the consumption bundle to maximize total utility is 2 chicken sandwich and 6 French fries, and \$0 left.

$$MRS_{FC} = \frac{MU_F}{MU_C} = \frac{5}{10} = \frac{1}{2}$$