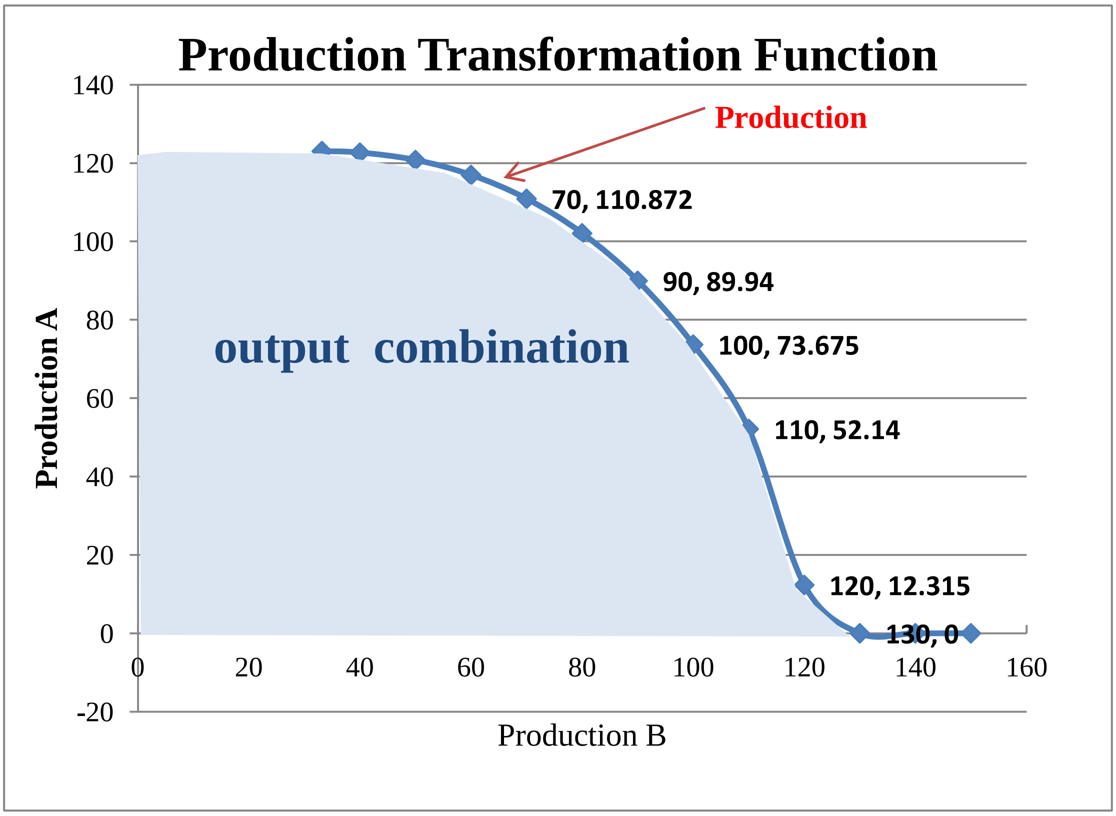
APEC 8202
HW3 Zhiyu Wang

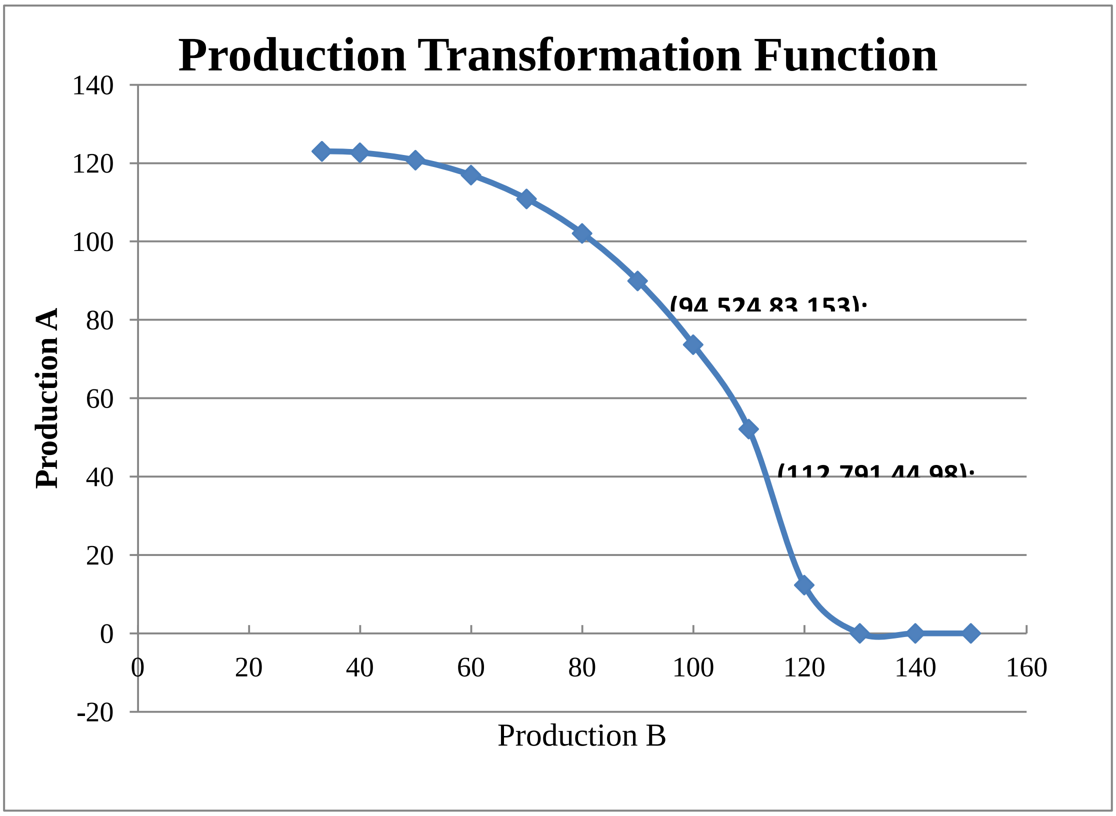
F. 1



From the graph we can find that as the production on one good increase, that on the other good decreases and it decreases at a increasing speed. On the other hand, we can say that the opportunity cost of production is increasing as the amount of product increases.

F.2. When PA=1.5 and PB=4, the revenue-maximizing solutions are YA=44.98, YB=112.791. When PA=2.5 and PB=4, the revenue-maximizing solutions are YA=83.153, YB=94.524

We can see that the revenue maximizing solutions are on the productivity possibility frontier. As price of A increases, the optimal solution to revenue maximization of product B inclines to go up.



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G.

G.1 Formulate the maximization problem of net revenue.

First, define all the variables.

Y(A)-sales of product A, Y(B)-sales of product B, Y(C)-sales of product C,
I(1)-amount of input 1, I(2)-amount of input 2, I(3)-amount of input 3,
I(4)-amount of input 4, I(5)-amount of input 5, I(C)-amount of C as an input,
T(A1)-amount of units on technique 1 producing A
T(A2)-amount of units on technique 1 producing A
T(A3)-amount of units on technique 1 producing A
T(B1)-amount of units on technique 1 producing B
T(B2)-amount of units on technique 1 producing B
T(B3)-amount of units on technique 1 producing B
T(C1)-amount of units on technique 1 producing C
T(C2)-amount of units on technique 1 producing C
T(C3)-amount of units on technique 1 producing C
P(A)-price of product A, P(B)-price of product B, P(C)-price of product C,
P(1)-price of input 1, P(2)-price of input 2,

The maximization problem on net revenue is

$$\max \sum_{i=A,B,C} P(i) * Y(i) - \sum_{i=1,2} P(i) * I(i)$$

s.t. For products:

$$\text{for each i} = A, B, Y(i) \le \sum\nolimits_{j=T(i1,i2,i3)} T(j)$$

$$\text{for product } C, Y(C) \le \sum_{j=T(C1,C2,C3)} T(j) - I(C)$$

For inputs:

$$0.4 * T(A1) + 0.6 * T(A2) + 1.2 * T(A3) + 1.1 * T(B1) + 1.4 * T(B2) + 2 * T(B3) + 0.1 * T(C2) + 0.4 * T(C3) \le I(1)$$

$$1.5 * T(A1) + 0.6 * T(A2) + 0.2 * T(A3) + 1.9 * T(B1) + 1.4 * T(B2) + 1.2 * T(B3) + 0.4 * T(C1) + 0.1 * T(C2) \le I(2)$$

$$0.5 * T(B1) + 0.5 * T(B2) + 0.5 * T(B3) + 0.2 * T(C1) + 0.2 * T(C2) + 0.2 * T(C3) \le I(3)$$

$$1.9 * T(A1) + 1.1 * T(A2) + 0.7 * T(A3) \le I(4)$$

$$0.4 * T(A1) + 0.8 * T(A2) + 1.6 * T(A3) \le I(5)$$

$$0.2 * T(A1) + 0.4 * T(A2) + 0.6 * T(A3) \le I(C)$$

For input constraints:

$$I(3) \le 390, I(4) \le 285, I(5) \le 225$$

All the variables above are non-negative.

G2.

```
OPTION LIMCOL=0, LIMROW=0;

SET J PRODUCTS /A,B,C/;
SET I INPUTS /1,2,3,4,5,C/;
SET T TECHNIQUES /TECHA1,TECHA2,TECHA3,TECHB1,TECHB2,TECHB3,TECHC1,TECHC2,
TECHC3/;
SET IVAR(I) VARIABLE INPUTS /1,2/;
SET IFIX(I) FIXED INPUTS /3,4,5,C/;

TABLE A(J,T) UNIT OUTPUT COEFFICIENTS
        TECHA1    TECHA2     TECHA3    TECHB1     TECHB2    TECHB3    TECHC1
TECH
C2    TECHC3
A      1          1          1
B                                    1          1          1
C                                                                       1
1
          1 ;

TABLE D(IVAR,T) UNIT VARIABLE INPUT REQUIREMENTS
        TECHA1    TECHA2     TECHA3    TECHB1     TECHB2    TECHB3    TECHC1
TECH
C2    TECHC3
 1        0.4         0.6             1.2         1.1            1.4          2.0
0.
1       0.4
 2       1.5         0.6            0.2         1.9           1.4         1.2       0.4
0.
1              ;

TABLE E(IFIX,T) UNIT FIXED INPUT REQUIREMENTS
        TECHA1    TECHA2     TECHA3    TECHB1     TECHB2    TECHB3    TECHC1
TECH
C2    TECHC3
3                                          0.5          0.5         0.5       0.2
0.
2       0.2
4      1.9       1.1          0.7
5      0.4       0.8          1.6
```

```
C      0.2       0.4        0.6
          ;

PARAMETER B(IFIX) FIXED INPUT ENDOWMENTS
/3 390, 4 285, 5 225, C 100000000/;

PARAMETER P(J) PRODUCT PRICES /A 3.1, B 4.1, C 0.88/;
PARAMETER R(IVAR) VARIABLE INPUT PRICES /1 1.88, 2 1.5/;

VARIABLES  ZP TOTAL PROFIT,
           Y(J) SALES,
           X(T) PRODUCTION ACTIVITIES,
           Z(IVAR) VARIABLE INPUT PURCHASES;

POSITIVE VARIABLES X(T), Z(IVAR);

EQUATIONS PROFIT NET PROFIT,
          OUTPUT(J) OUPUT,
          VARINPUT(IVAR) VARIABLE INPUT USE,
          FIXINPUT(IFIX) FIXED INPUT USE,
          CONSTR_C CONSTRANT ON INPUT C;

PROFIT.. ZP=E=SUM(J,P(J)*Y(J))-SUM(IVAR,R(IVAR)*Z(IVAR));
OUTPUT(J).. Y(J)-SUM(T,A(J,T)*X(T))=L=0;
VARINPUT(IVAR).. SUM(T,D(IVAR,T)*X(T))-Z(IVAR)=L=0;
FIXINPUT(IFIX).. SUM(T,E(IFIX,T)*X(T))=L=B(IFIX);
CONSTR_C.. SUM(T,E("C",T)*X(T))+Y("C")=L=SUM(T,A("C",T)*X(T));

MODEL PRODA1 /ALL/;
SOLVE PRODA1 USING NLP MAXIMIZING ZP
```

```
COMPILATION TIME     =        0.000 SECONDS      3 Mb  WEX239-239 Aug 29, 2012
```

```
MODEL STATISTICS

BLOCKS OF EQUATIONS           5     SINGLE EQUATIONS           11
BLOCKS OF VARIABLES           4     SINGLE VARIABLES           15
NON ZERO ELEMENTS            58     NON LINEAR N-Z              0
DERIVATIVE POOL              10     CONSTANT POOL              16
CODE LENGTH                   0


GENERATION TIME      =        0.016 SECONDS      4 Mb  WEX239-239 Aug 29, 2012


EXECUTION TIME       =        0.016 SECONDS      4 Mb  WEX239-239 Aug 29, 2012
```

```
               S O L V E      S U M M A R Y

     MODEL   PRODA1              OBJECTIVE  ZP
     TYPE    NLP                 DIRECTION  MAXIMIZE
     SOLVER  CONOPT              FROM LINE  58

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      1 Optimal
**** OBJECTIVE VALUE             1243.4455

 RESOURCE USAGE, LIMIT          0.000      1000.000
 ITERATION COUNT, LIMIT         5    2000000000
 EVALUATION ERRORS              0             0
CONOPT 3         Jul  4, 2012 23.9.2 WEX 34973.35015 WEI x86_64/MS Windows


    C O N O P T 3   version 3.15F
    Copyright (C)   ARKI Consulting and Development A/S
                    Bagsvaerdvej 246 A
                    DK-2880 Bagsvaerd, Denmark


 ** Optimal solution. There are no superbasic variables.


 CONOPT time Total                            0.001 seconds
   of which: Function evaluations             0.000 =  0.0%
             1st Derivative evaluations       0.000 =  0.0%


                       LOWER     LEVEL     UPPER    MARGINAL

---- EQU PROFIT          .         .         .        1.000

  PROFIT  NET PROFIT

---- EQU OUTPUT  OUPUT

     LOWER     LEVEL     UPPER    MARGINAL
```

```
A      -INF        .          .         3.100
B      -INF        .          .         4.100
C      -INF    -103.636      .          .

---- EQU VARINPUT  VARIABLE INPUT USE

     LOWER     LEVEL     UPPER    MARGINAL

1      -INF        .          .         1.880
2      -INF        .          .         1.500

---- EQU FIXINPUT  FIXED INPUT USE

     LOWER     LEVEL     UPPER    MARGINAL

C      -INF     103.636 1.0000E+8      .
3      -INF     390.000   390.000     2.710
4      -INF     285.000   285.000     0.655
5      -INF     207.273   225.000      .

                        LOWER     LEVEL     UPPER    MARGINAL

---- EQU CONSTR_C       -INF        .          .         0.880

  CONSTR_C  CONSTRANT ON INPUT C

                        LOWER     LEVEL     UPPER    MARGINAL

---- VAR ZP             -INF    1243.445     +INF        .

  ZP  TOTAL PROFIT

---- VAR Y  SALES

     LOWER     LEVEL     UPPER    MARGINAL

A      -INF     259.091     +INF        .
B      -INF        .        +INF        .
C      -INF    1846.364     +INF        .

---- VAR X  PRODUCTION ACTIVITIES

          LOWER     LEVEL     UPPER    MARGINAL
```

```
TECHA1       .          .        +INF     -1.322
TECHA2       .       259.091     +INF       .
TECHA3       .          .        +INF     -0.442
TECHB1       .          .        +INF     -2.173
TECHB2       .          .        +INF     -1.987
TECHB3       .          .        +INF     -2.815
TECHC1       .          .        +INF     -0.262
TECHC2       .      1950.000     +INF       .
TECHC3       .          .        +INF     -0.414

---- VAR Z  VARIABLE INPUT PURCHASES

     LOWER     LEVEL     UPPER    MARGINAL

1      .       350.455     +INF       .
2      .       350.455     +INF       .


**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
                             0     ERRORS


EXECUTION TIME       =        0.000 SECONDS      2 Mb  WEX239-239 Aug 29, 2012


USER: GAMS Development Corporation, Washington, DC   G871201/0000CA-ANY
      Free Demo,  202-342-0180,  sales@gams.com,  www.gams.com   DC0000


**** FILE SUMMARY

Input      F:\Study\APEC 8202\Lab\Exercise G.gms
Output     C:\Users\Zhiyu\Documents\gamsdir\projdir\Exercise G.lst
```

G3.

The optimal solution is that

T(A1)=T(A3)=T(B1)=T(B2)=T(B3)=T(C1)=T(C3)=0, T(A2)=259.091, T(C2)=1950,

the maximum of net revenue=1243.445,

Y(A)=259.091, Y(B)=0, Y(C)=1846.364,

I(1)=350.455, I(2)=350.455

Economic interpretation:

At the optimal point, firms will produce zero in product B due to its marginal profit is always lower than those of both product A and B. Thus, the amount on units of techniques that produce B is zero. When producing A, the marginal profit of technique 2 is always higher than technique 2 and 3, thus firms only use technique 2 to produce A. Similarly, firms only use technique 3 to produce C.