1. **Question 1**: Suppose that a number of people in the area live in Minneapolis and work in St. Paul. Each person has a choice of the commuting method: taking a bus (choice i), driving a car (choice j), or taking the newly built Light Rail (choice k). Your task is to model the choice between the three commuting alternatives, using the random utility model that Professor McCullough presented in class. That is, the utility from choosing an alternative i by a person n is
$$u_{in} = V_{in} + \varepsilon_{in}$$
where V_{in} is a set of characteristics of the alternative (i.e. waiting time, price of gasoline, price of parking, etc). Assume that $\varepsilon_{in}, \varepsilon_{jn}, \varepsilon_{kn}$ are Gumbel distributed so their difference has a logistic distribution:
$$F(\varepsilon_n) = \frac{1}{1 + \exp^{-\varepsilon_n}}$$
Using Professor McCullough's example of a model with two alternatives, derive the choice probability $P_n(i)$ for the logit model with three alternatives. Then, find the expression for the marginal effects of an explanatory variable X_i on the probability of choosing the alternative i, i.e. $\frac{\partial P_n(i)}{\partial X_i}$ (assume that X_i is a continuous variable).

2. **Question 2**: This problem is an adaptation of the Application 1 from Greene Chapter 14 (Chapter 16 in the 6th edition). It is based on the German Health Care data (an unbalanced panel) found in Table F7.1 in the Greene appendix (the shorter version with the relevant variables is available on the class website). We consider analysis of a dependent variable, y_{it}, that takes values of 1 and 0 with probabilities $F(\mathbf{x}_{it}'\boldsymbol{\beta})$ and $1 - F(\mathbf{x}_{it}'\boldsymbol{\beta})$, where F is a function that defines probability. The dependent variable y_{it} is constructed from the count variable *Docvis*, which is the number of visits to a doctor in the given year. Construct the binary variable
$$y_{it} = 1 \text{ if } Docvis_{it} > 0, \text{ 0 otherwise}$$
We will build a model for the probability that y_{it} equals 1. The independent variables of interest will be
$$\mathbf{x}_{it} = (1, age_{it}, educ_{it}, female_{it}, married_{it}, hsat_{it})$$
(See Greene Appendix Table F7.1 for the description of the variables).

   (a) According to the model, the theoretical density for y_{it} is:
   $$f(y_{it} \mid \mathbf{x}_{it}) = F(\mathbf{x}_{it}'\boldsymbol{\beta}) \text{ for } y_{it} = 1 \quad\text{and}\quad 1 - F(\mathbf{x}_{it}'\boldsymbol{\beta}) \text{ for } y_{it} = 0$$
   We will assume that a logit model is appropriate, so that
   $$F(\mathbf{x}_{it}'\boldsymbol{\beta}) = \Lambda(\mathbf{x}_{it}'\boldsymbol{\beta}) = \frac{\exp(\mathbf{x}_{it}'\boldsymbol{\beta})}{1 + \exp(\mathbf{x}_{it}'\boldsymbol{\beta})}$$
   Show that for the two outcomes, the probabilities may be combined into the density function
   $$f_{it}(y_{it} \mid \mathbf{x}_{it}) = g(y_{it}, \mathbf{x}_{it}, \boldsymbol{\beta}) = \Lambda[(2y_{it} - 1)\mathbf{x}_{it}'\boldsymbol{\beta}]$$
   Use this result to construct the log-likelihood function for a sample of data on $(y_{it}, \mathbf{x}_{it})$. Note that in this application we ignore the panel aspect of the data set—build the model as if this were a cross section.

   (b) Derive the likelihood equations for estimation of $\boldsymbol{\beta}$, i.e. $\frac{\partial \ln L}{\partial \beta}$.

(c) Derive the second derivatives matrix of the log likelihood function (Hint: the following will prove useful in the derivation: $d\Lambda(t)/dt = \Lambda(t)[1 - \Lambda(t)]$).

(d) Obtain maximum likelihood estimates of the parameters for the data and the variables noted. Report your parameter estimates and standard errors, as well as the value of the likelihood function. How do you interpret the coefficients?