Applied Economics 8602
Economics of the Environment
Practice problems for mid-term exam

These problems are like those you will encounter on the take-home exam next week. I've gleaned them from old exams, excluding those that are related to material we haven't yet covered.

1. (*Public Goods.*) An economy consists of two consumers, each of whom consumes two goods: a public good q and a private good x_j. Consumer 1's utility function is $U_1(x_1, q) = x_1 q$ and consumer 2's utility function is $U_2(x_2, q) = x_2 + q$. Let $\omega_j = 10$ be j's endowment of good x, of which the amount $z_j \in [0, 10]$ is devoted to the provision of the public good, whose production technology is described by $y = (z_1 + z_2)/2$. The remainder of j's endowment is consumed directly as good x. (That is, $x_j + z_j = \omega_j$.) Determine the Pareto-optimal allocation for this economy. (*Hint:* You may want to use the fact that with linear utility, we can't be sure that the solution is interior.)

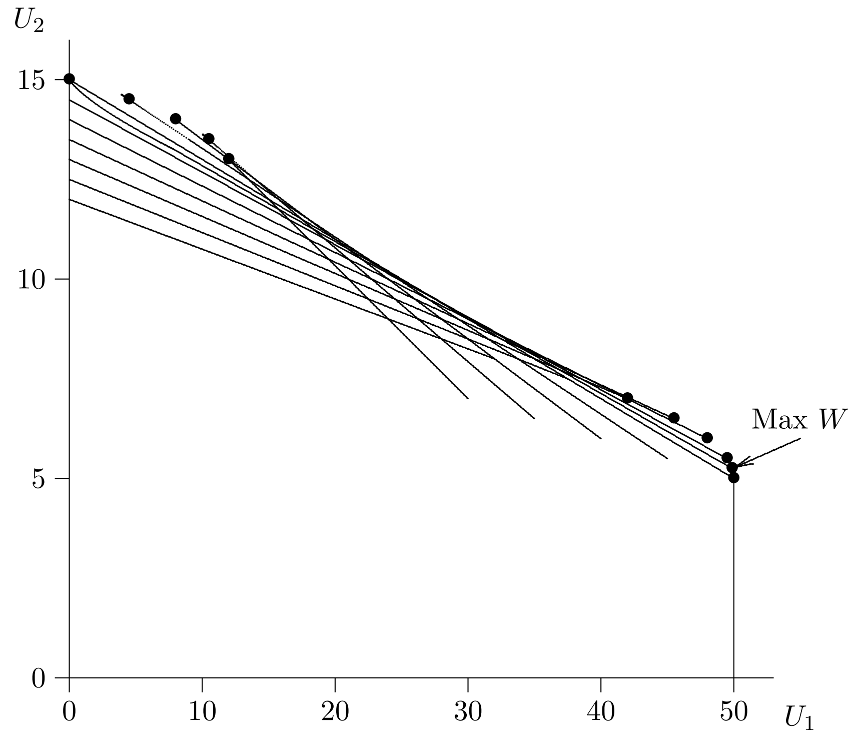
   Solution. The difficulty with this problem is that the PO solutions that are easiest to find are corner solutions. One may set up the social planner's objective function as follows, substituting the resource constraint for each person and the technology into the respective utility functions:

$$\begin{aligned}\max_{z_1, z_2} W &= U_1 + U_2 \\ &= (10 - z_1)\left(\frac{z_1 + z_2}{2}\right) + (10 - z_2) + \left(\frac{z_1 + z_2}{2}\right).\end{aligned}$$

   The FONCs for this problem are

$$\begin{aligned}\frac{\partial W}{\partial z_1} &= 5 - z_1 - z_2/2 + 0.5 = 0 \\ \frac{\partial W}{\partial z_2} &= 5 - z_1/2 - 1 + 0.5 = 0.\end{aligned}$$

   If one attempts to solve these expressions for the z_i^*, one finds that the solution involves a negative value for z_2. The second FONC yields $z_1^* = 9$. Plugging this into the second yields $z_2^* = -7$, which is of course infeasible. Those who tried this problem used various strategies to reach a solution in the face of this difficulty. What I had hoped that you would do is try to find an outcome that maximizes either U_1 or U_2. These will be PO by definition. The highest level that U_2 can ever take is achieved when $z_1 = 10$ and $z_2 = 0$. This gives a utility pair of $(0, 15)$. The highest level that U_1 can ever take is achieved when $z_1 = 0$ and $z_2 = 10$. This gives a utility pair of $(50, 5)$. Both of these outcomes are Pareto optimal. It turns out that the highest possible level of $W = U_1 + U_2$ is acheived when $z_1 = 0.5$ and $z_2 = 10$. This gives a utility pair of $(49.875, 5.25)$. It takes some computation to find this solution, so I didn't expect anyone to find it. The following diagram illustrates the frontier. Each of the individual lines in the diagram represents the possible utility outcomes for a given level of $z_1 + z_2$. The dots are points on the frontier.



2. (*Public bads.*) An economy consists of 2 identical agents and two goods, one private good x and one public *bad* y. Consumers have utility functions $U_j(x_j, y) = 2x_j - .25y$, and identical endowments of the private good, ω_j. Consumption of x produces y on a one-for-one basis, except that abatement activity may be undertaken to reduce y. Let j's contribution to abatement effort be given by $a_j = \omega_j - x_j$. Production of y is given by $y = x_1 + x_2 - k(a_1 + a_2)$, where $k \in (0, 1)$ is a constant. At a Pareto optimum for this economy, how much of aggregate ω will be devoted to abatement activity? (*Hints:* Use the fact that the consumers are identical. Consider the possibility of a corner solution.)

   Solution. This problem is linear, so one can't rely on the existence of an interior solution. Indeed, the solution is to spend nothing at all on abatement. One way to see this is to note that eating a unit of ω gives our consumers 2 utils from x and $-.25$ utils from the associated y. The problem is somewhat more complex, though, because we have two people and the mapping between x and y involves the parameter k. (But note that the effect of $k < 1$ pushes things in favor of eating everything, because it reduces the externality effect of increasing consumption.) Further, person 2's consumption of x causes an externality for person 1, and vice versa. The social planner is not concerned with strategic behavior, so one can use the fact that the two agents are identical, ignoring individual strategic incentives altogether. This is not a Nash problem. Simply write x for x_j, a for a_j, and ω for ω_j. The social planner's problem may be written

$$\max_{x,y} \quad U_1 + U_2 = 2(2x - .25y) \quad \text{s.t.} \quad y = 2x - 2ka.$$

Substituting the constraint into the objective function, and now using $a = a_1 + a_2$, we have

$$\begin{aligned}\max_x \quad U_1 + U_2 &= 4x - .5(2x - 2ka) \\ &= 4x - .5(2x - 2k(2\omega - 2x)) \\ &= 3x - 2kx + 2k\omega \\ &= (3 - 2k)x + 2k\omega.\end{aligned}$$

This function is linear and increasing in x so long as $k < 3/2$, which it is. Thus, the solution is at the largest possible x, which is 2ω, or $x_j = \omega_j$.

3. (*Permit trading and banking.*) Two firms are the only source of a pollutant in their airshed. The firms are currently unregulated. Firm 1 emits $e_1^0 = 50$ tons; firm 2 emits $e_2^0 = 100$. The firms' abatement cost functions are $C_1(a_1) = a_1^2/2$ and $C_2(a_2) = a_2^2/4$. A regulator has decided that emissions should be reduced by 60%, from 150 to 60. She wishes to use a permit-trading scheme to achieve this reduction. Firm 1 is granted $\ell_1^0 = 20$ and firm 2 is granted $\ell_2^0 = 40$ permits, at zero cost, and the two are allowed to trade permits.

   a. Find the equilibrium distribution of permits, the emissions level for each firm, the aggregate cost of abatement, and the permit price. Be sure to state clearly any assumptions that you adopt in order to reach your solution.

   Solution. I like to transform the problem so that everything depends on the $e_i = e_i^0 - a_i$. If we assume that firms take prices as given, then firm i solves

   $$\min_{e_i} C_i(e_i) + p^\ell(e_i - \ell_i).$$

   The FONCS yield the following expressions, which are demand functions for permits:

   $$\begin{aligned}e_1(p) &= 50 - p \\ e_2(p) &= 100 - 2p.\end{aligned}$$

   Setting the left sides equal and using the fact that $e_1 + e_2 = 60$, we obtain $e_1^* = 20$ and $e_2^* = 40$. Thus, $a_1^* = 30$ and $a_2^* = 60$. The equilibrium permit price is $p^* = 30$. No permit trades take place: $e_i^* = \ell_i$ for each i. Abatement costs are $C_1 = 450$ and $C_2 = 900$, for a total of $C = 1350$.

   b. Suppose now that the problem lasts for two periods and the discount rate is 10%. Firms are given the same quantities of permits, ℓ_i^0, as in part a., only now they receive the grant each period. They may bank or borrow permits between periods, so long as the sum of aggregate emissions over the two periods dos not exceed 120. *True or false:* Abatement in the first period of this 2-period model will be lower than in the 1-period model of part a.

   Solution. You did not need to prove anything here. It was enough to note that, as in the Kling and Rubin work, banking and borrowing usually leads to borrowing: firms have an incentive to delay their abatement expenditures so that the present value of costs is minimized. An exception to this result would be if, for example, the marginal cost of abatement falls over time, with the rate of decrease sufficiently large relative to the discount rate.

4. (*Prices versus quantities.*) A polluting industry consists of two firms that are the only sources of a perfectly mixed pollutant. Before controls are put in place, the firms emit $e_1 = 100$ and $e_2 = 150$. Abatement is costly. The firms' abatement cost functions are

$$C_1(q_1) = 3q_1 + q_1^2/4 \quad \text{and}$$
$$C_2(q_2) = 3q_2 + q_2^2/8,$$

where q_i is abatement by firm i. An environmental regulator has decided that emissions should be reduced. The regulator's assessment of abatement benefits is given by

$$E(B(q)) = 45q - q^2/6 + u,$$

where $q = q_1 + q_2$ and $u \sim D(0, \sigma^2)$ is a mean-zero error term.

a. Determine the optimal level of the tax that a *tax-setting* regulator should choose. From this, calculate each firm's abatement level.

Solution. Industry MAC is found by summing horizontally: $q_1 = 2p - 6$ and $q_2 = 4p - 12$, so $q = 6p - 18$. Inverting the sum, $MC = 3 + q/6$. Set this equal to E(MB) = 45 - q/3 to obtain $q^* = 84$ and $t^* = 17$. Individual abatement levels will be $q_1^* = 28$ and $q_2^* = 56$. Emissions are $e_1^* = 100 - 28 = 72$ and $e_2^* = 150 - 56 = 94$, so aggregate emissions are $e^* = 128$.

b. Now suppose the regulator wishes to impose a *permit-trading* scheme to achieve the same level of aggregate abatement. Determine how many permits the regulator will choose to allocate to each of the two firms. If the firms behave competitively in the allowance market, what will be the equilibrium price? How many allowances will be traded?

Solution. To achieve the same result, the regulator should issue 166 permits. The distribution of permits is irrelevant to the ultimate outcome in this case. The firms will emit the same amounts as in part a.

c. Which policy approach, taxes or permits, will maximize expected social welfare? Justify your answer carefully.

Solution. Expected welfare is found by integrating the difference between marginal benefit and aggregate marginal cost, from zero up to some large value of q. The solution is $E(W) = 1764$ and is the same whether permits or taxes are used.

5. (*Technology adoption.*) Two identical firms are the only source of a given pollutant. Each currently emits $E_i = 80$ units of the pollutant, and initial abatement costs for each firm are $C_0^i(q^i) = (q^i)^2$. The firms have available an improved abatement technology, at a cost of $F_i = 112.5$, that reduces abatement costs to $C_I^i(q^i) = (2/3)(q^i)^2$. Marginal benefits to abatement, experienced by society at large, are $\text{MB}(q) = 80 - q$, where $q = q^1 + q^2$.

a. A regulator enters the picture and decides to apply a Pigouvian tax t to each unit of emissions. *If the regulator knows that the low-cost abatement technology is available*, determine the tax level at which the firms are indifferent between adopting the new technology and not adopting it.

Solution. A firm using the high-cost technology will choose q_0^i as a function of t to minimize $\text{TC}_0(q_0^i) = (q_0^i)^2 + t(E^i - q_0^i)$, with FONC $\text{TC}_0' = 2q_0^i - t = 0$, or $q_0^i(t) = t/2$. A firm using the low-cost technology will minimize $\text{TC}_I' = (2/3)\cdot(q_I^i)^2 + t(E^i - q_I^i) + 112.5$, with FONC $\text{TC}_I^i = (4/3)q_I^i - t = 0$, or $q_I^i(t) = (3/4)t$. Set $\text{TC}_0^i = \text{TC}_I^i$ and solve for t:

$$(t/2)^2 + t(E - (t/2)) = (2/3)((3/4)t)^2) + t(E - (3/4)t) + 112.5,$$

or

$$t^2\left(\frac{1}{4} - \frac{1}{2} - \frac{3}{8} + \frac{3}{4}\right) = 112.5,$$

which yields $t^2 = 900$ or $t^* = 30$. For any $t > 30$, the firms will adopt the new technology.

b. Suppose now that *the regulator is unaware of the opportunity firms face* to purchase the low-cost abatement technology,believing (mistakenly) that the only technology available is the high-cost technology. Determine the tax that such a regulator would impose in order to maximize perceived social welfare, given by the difference between aggregate abatement costs and aggregate benefits. How much will firms abate and what is actual social welfare under this scenario?

Solution. Aggregate MC under the high-cost technology is $\text{MC}_0(q_0) = q_0$. Set this equal to marginal benefits to obtain $80 - q_0 = q_0$, or $q_0^* = 40$. The required tax is $t^* = 40$. At this tax rate, which exceeds the threshold value from part a. above, both firms will adopt the new technology. In response to the tax rate $t = 40$, each firm will actually abate $q_0^i = 30$, more than the regulator expected, so that $q_0 = 60$. Actual social welfare is

$$\begin{aligned} W &= \int_0^{60} [(80 - q) - (2/3)q]\, dq - 225 \\ &= (80q - q^2/2 - q^2/3) \mid_0^{60} - 225 \\ &= 1575. \end{aligned}$$

c. What would be the gain in social welfare if the regulator *was aware of the firms' opportunity to adopt the low-cost technology*? Compute the optimal tax, abatement levels, the technology-adoption decision, and the welfare gain relative to your result in part b.

Solution. The regulator now recognizes that the firms will respond by choosing optimally whether to adopt the new technology, and also by setting MC equal to the tax along the relevant MC curve. If firms adopt the new technology, the regulator should choose a tax so that $\text{MC}(q_I) = (2/3)q_I = \text{MB}(q)$. Solving, we find that $q_I^* = 48$, so that $t_I^* = 32$. At this tax rate both firms will indeed adopt the new technology. Social welfare is now

$$\begin{aligned} W &= \int_0^{48} [(80 - q) - (2/3)q]\, dq - 225 \\ &= (80q - q^2/2 - q^2/3) \mid_0^{48} - 225 \\ &= 1695. \end{aligned}$$

The welfare gain relative to the outcome for an uninformed regulator is $\Delta W = 1695 - 1575 = 120$.

6. (*Public goods.*) Two consumers are the only members of an island economy. They have identical preferences over two goods, a private numeraire good x and a pure public good q. Preferences are given by

$$U_i(x_i, q) = \ln x_i + 2 \ln q.$$

Each consumer is endowed with $\omega_i = 10$ of the private good, of which x_i is consumed directly and the remainder $z_i = \omega_i - x_i$ is contributed to the provision of the public good. q is produced according to the simple production function $q = z_1 + z_2$.

a. Determine the outcome (x_1, x_2, z_1, z_2), that a benevolent social planner would choose so as to maximize the unweighted sum of preferences, $W = U_1(x_1, q) + U_2(x_2, q)$.

Solution. Exploiting the fact that the consumers are identical, the planner's problem may be written

$$\max_x 2 \ln x + 4 \ln q$$
$$\text{s.t.} \quad q = 20 - 2x,$$

or

$$\max_x \ 2 \ln x + 4 \ln(20 - 2x),$$

with FONC

$$\frac{2}{x} = \frac{4}{10 - x}.$$

This may be rearranged as

$$4x = 20 - 2x,$$

so that $x_i^* = 10/3$ and $z_i^* = 20/3$ for an optimal level of the public good $q^* = 40/3$.

b. Show that if z_2 is held fixed at the solution you found in part a., consumer 1's best response is to contribute less than z_1^*.

Solution. Consumer 1 chooses x_1 to solve

$$\max_x \ U_1(x_1; z_2) = \ln x_1 + 2 \ln(10 - x_1 + (20/3)),$$

with FONC

$$\frac{\partial U_1}{\partial x_1} = \frac{1}{x_1} - \frac{2}{(50/3) - x_1} = 0.$$

This yields $\hat{x}_1 = 5.555$, and thus $\hat{z} = 4.444 < 20/3$.

c. Find the Lindahl price at which the consumers, taking this price as given when selecting their contribution z_i, will choose the socially optimal level of contribution to the public good.

Solution. Given that the price of x is 1 and consumers must pay p_i for q, each consumer solves $\max_{x_i,q} \ln x_i + 2\ln q$ subject to $10 = x_i + p_i q$, with FONCs

$$\frac{1}{x_i} - \lambda = 0 \quad \text{and} \quad \frac{2}{q} - \lambda p_i = 0.$$

This gives $1/x_i = 2/p_i q$, or $x_i = q \cdot (p_i/2)$. Plug this into the constraint to get

$$q(p_i) = \frac{20}{3p_i}.$$

This is i's demand for q. Demand for x_i is then

$$x_i(p_i) = \frac{p_i}{2}\frac{20}{3p_i} = \frac{10}{3}.$$

We want to know what p_i should be in order to induce i to "purchase" exactly $q = 40/3$. The solution is $p_i^* = 1/2$. A producer facing these Lindahl prices and purchasing z at a price of 1 will solve $\max(p_1 + p_2)q - z$ subject to $z = q$. The solution is that $p_1 + p_2 = 1$, which agrees with the outcome for consumers.

7. (*Permit trading.*) Two competitive firms are the only sources of sulfur dioxide in their airshed. Firm 1 currently emits $e_1 = 2000$ tons of SO_2 and firm 2 emits $e_2 = 1000$. An environmental regulator has decided that SO_2 emissions must be reduced to 2000 tons. The firms' abatement cost functions are given by

$$C_1(a_1) = .02a_1^2 \quad \text{and} \quad C_2(a_2) = .04a_2^2$$

respectively. The regulator has decided to implement a permit-trading scheme in order to reach the emissions target. Under this scheme, each firm is endowed with $\ell_i = 1000$ permits at zero cost, where a permit gives its owner the right to emit one ton. The firms may then trade permits with each other, but each must emit no more tons of SO_2 than the number of permits it holds.

   a. (*10 points.*) Write down the optimization problem that each firm will solve. Define an equilibrium in the permit market.

      Solution. Firm i solves $\min_{a_i} C_i(a_i) + p_\ell(\bar{e}_i - a_i - \ell_i)$. An equilibrium is a vector (a_1^*, a_2^*, p_ℓ^*) at which both firms optimize and markets clear.

   b. (*15 points.*) Suppose the permit market is perfectly competitive. Determine the equilibrium outcome in the permit market, including each firm's emissions level and abatement costs, as well as the permit price.

      Solution. The equilibrium price is $p_\ell^* = 26.67$ and equilibrium abatement levels are $a_1^* = 666.67$ and $a_2^* = 333.33$. Firm 1 buys 333.33 permits from firm 2. Abatement costs are $C_1(a_1^*) = 8888.89$ and $C_2(a_2^*) = 444.44$.

   c. (*10 points.*) Now suppose that each firm recognizes that it is the only participant on its side of the market. If the buyer of permits behaves behaves as a monopsonist and the seller as a monopolist, what will be the outcome of the trading scheme? Explain your answer, including the distribution of abatement and the sharing of rents, carefully.

Solution. Several responses were accepted here. My preferred response was that, because there's no Nash equilibrium to a bilateral-monopoly problem, the two firms would collude to set abatement levels optimally and then bargain over the rents. Some laid out and solved a Stackelberg problem in which one firm is able to extract the rents from the other.

8. (*Prices and quantities.*) Consider the problem facing an environmental regulator who wishes to control emissions of a single pollutant, emitted by a single competitive industry. Emissions are currently uncontrolled. Aggregate industry marginal abatement costs (MAC) as a function of abatement q are given by $MAC = a + bq + u$, where u is distributed uniformly on the interval $[-\beta, \beta]$. Social marginal benefits from abatement, known with certainty, are given by $MB = c - dq$. All of the parameters, a, b, c, and d, are strictly positive.

   a. (*10 points.*) If the regulator's goal is to control pollution so as to maximize expected social welfare, find an expression for the optimal level of abatement.

   Solution. $q^* = (c - a)/(b + d)$.

   b. (*15 points.*) Write down a set of conditions on this problem sufficient to guarantee that a price (or tax) instrument is preferred to a quantity (or permit) instrument. A diagram may prove helpful, but you should provide a careful proof establishing your result.

   Solution. A tax is preferred if marginal cost is steeper than marginal benefits: $b > d$.

   c. (*10 points.*) If $a = 5$, $b = 0.5$, $c = 40$, $d = 1.0$, and $\beta = 5$, compute the difference in expected deadweight loss between the price instrument and the quantity instrument. (Your result may be negative or positive.)

   Solution. Basing her policy on $E(u) = 0$, the regulator will set the tax at $t = 16.67$. For a given realization of u, calculate the deadweight loss under a tax scheme as follows. If the regulator knew the true cost function, she would set the tax to achieve abatement q^u so that $5 + .5q + u = 40 - q$. This gives $q^u(u) = (35 - u)/1.5$. Given the tax, though, the industry will emit so that actual MC equals 16.67. That is, the industry solves $5 + .5q + u = 16.67$. This gives $q^a(u) = 23.33 - 2u$. Deadweight loss under a tax is the area of a triangle on its side, with its base a vertical distance and its height a horizontal distance in the relevant diagram. The base is the difference between MB and actual MC for any given u. This difference is

$$\begin{aligned}
\text{Base}(u) &= \mid \text{MB}(q^a(u)) - \text{MC}(q^a(u)) \mid \\
&= \mid 40 - q^a - (5 + .5q^a + u) \mid \\
&= \mid 40 - 23.33 + 2u - 5 - .5(23.33 - 2u) + u \mid \\
&= \mid 2u \mid .
\end{aligned}$$

   The height of the triangle is the horizontal difference between the optimal q and the abatement quantity chosen by the industry facing a tax of 16.67. This is

$$\begin{aligned}
\text{Height}(u) &= \mid q^u(u) - q^a(u) \mid \\
&= \left| \frac{35-u}{1.5} - (23.33 - 2u) \right| \\
&= \mid 1.33u \mid .
\end{aligned}$$

The area of the triangle is one half its base times its height, or

$$\text{DWL}^t(u) = 1.33u^2.$$

The expected value of the deadweight loss, then, is the integral of this expression over the relevant uniform density $f(u) = 1/10$ and support on u:

$$\begin{aligned}
E(\text{DWL}^t) &= \int_{-5}^{5} 1.33u^2 f(u)\, du = \int_{-5}^{5} \frac{1.33u^2}{10}\, du \\
&= \left. \frac{.133u^3}{3} \right|_{-5}^{5} = 11.11.
\end{aligned}$$

Calculate the deadweight loss under a permit scheme as follows. The regulator will issue permits so that abatement must be $q = 23.33$. If she knew the true cost function, she would issue permits so that abatement is q^u. The deadweight loss triangle here has a base (in the vertical direction) of $\mid u \mid$ and a height (in the horizontal direction) of $\mid 23.33 - q^u \mid = \mid .667u \mid$. The area of the triangle is

$$\text{DWL}^\ell(u) = 0.333u^2.$$

The expected value of the deadweight loss, then, is the integral of this expression over the relevant uniform density $f(u) = 1/10$ and support on u:

$$\begin{aligned}
E(\text{DWL}^\ell) &= \int_{-5}^{5} 0.333u^2 f(u)\, du = \int_{-5}^{5} \frac{0.333u^2}{10}\, du \\
&= \left. \frac{.0333u^3}{3} \right|_{-5}^{5} = 2.778.
\end{aligned}$$

The difference between these two numbers is the correct answer to the question given:

$$E(\text{DWL}^t) - E(\text{DWL}^\ell) = 11.111 - 2.778 = 8.333 > 0.$$

The permit scheme is preferred.