1. Consider a cash credit economy with endogenous labor supply in which at all dates except some T, money growth is μ. At $T \geq 1$, money growth is $\mu_T > \mu$. Assume preferences are $\log c_1 + \alpha \log c_2 + \gamma \log(1 - \ell)$. The resource constraint is $c_1 + c_2 = \ell$. Let $T = 2$. Characterize an equilibrium which is stationary after date T.

2. Prove that in a cash-credit goods economy with constant money growth $\mu > \beta$, if the economy is in a stationary equilibrium from some T onwards, it is stationary at all previous dates.

3. Consider a cash-credit goods economy with preferences given by $\log c_{1t} + \alpha \log c_{2t} + \gamma \log(1 - n_t)$ where c_{it} denotes consumption of cash goods, c_{2t} denotes credit goods consumption, n_t denotes time allocated to market. The resource constraint is

$$c_{1t} + c_{2t} = n_t.$$

The cash-in-advance constraint is
$$p_t c_{1t} \leq M_t,$$

where p_t denotes the price level and M_t denotes cash balances. Households trade money and bonds in the securities market at the beginning of each period. The securities market constraint is

$$M_t + B_t \leq (M_{t-1} - p_{t-1} c_{1t-1}) - p_{t-1} c_{2t-1} + w_{t-1} n_{t-1} + R_{t-1} B_{t-1} - T_t,$$

where B_t denotes holdings of one-period bonds, R_t the interest rate, w_t the wage rate and T_t lump-sum taxes. The government conducts monetary policy to keep the interest rate fixed at some level R for all periods.

   (a) Define a competitive equilibrium.

   (b) What happens to n_t as R increases? Prove your result.

   (c) Now suppose that the interest rate is fixed at R^0 in even periods and $R^1 > R^0$ in odd periods. Compute equilibrium allocations in the two types of periods. What can you say about money growth from odd to even periods and from even to odd periods?

4. Show that a cash-credit goods economy in the utility function are equivalent (in the sense that one can relabel goods). That is, let $c_t = c_{1t} + c_{2t}$, and let real balances at the end of the period in a money in the utility function economy equal c_{1t}.

5. Set up a stochastic version of Lucas-Stokey's cash-credit goods model with endogenous labor supply. Assume the only randomness is in the quantity of money. Derive sufficient conditions on the stochastic process governing money growth to ensure that the cash-in-advance constraint is binding on all states in a stationary equilibrium. (Assume at least 2 states with different money growth rates in each). Assume the utility function is separable in the two goods and leisure. Suppose money growth can take on one of two values and that the probability money growth is high tomorrow is increasing on today's money growth. Derive sufficient conditions for output to be low in the high money growth rate state.

6. Show that the price level is indeterminate on a cash-credit goods model with Lucas-Stokey timing if the policy is specified as an exogenous sequence of interest rates.(hint: as shown in class, same allocation can be associated to different prices and policies. Make the statement formal and prove it.)

7. Read 'Huggett-The risk free rate in heterogeneous-agent incomplete-insurance economies' and 'Aiyagari-Uninsured idiosyncratic risk and aggregate saving'. Identify assumptions in each of the papers that allow them to use theorem 12.12 in SLP to prove the existence of and convergence to an invariant distribution. (hint: focus on monotonicity; feller property and mixing condition.)