APEC 8212 Econometric Analysis II: Recitation Week 11

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1 Difference in differences (DID)

Consider a program starts in year $t = 1$ for a group labeled by $w = 1$. The object of the program is some value $y = y_0 + w(y_1 - y_0)$, where y_1 is the outcome with the program and y_0 is the outcome without the program. We can only observe y_1 for any observation in the program. To evaluate the effect of program τ, we need to use a control group labeled by $w = 0$.

The idea of DID is that instead of comparing the difference of y between program group $w = 1$ and control group $w = 0$, the effect of prgram τ can be also estimated by comparing the difference of y of program group $w = 1$ between before the program ($t = 0$) and after the program ($t = 1$). However, it's always possible that, besides the program, the change of y overtime is also caused by some time trend. To solve this problem, suppose the control group has the same time trend, then we can first estimate the time trend using the data of control group, then subtract it from the cross-time difference of y of the program group. To do this, we can run the following linear regression

$$y = \gamma_0 + \gamma_1 wt + \gamma_2 w + \gamma_3 t + u$$

The effect of program τ is estimated by γ_1.

The following table explains this idea clearer.

		$\mathrm{E}(y \mid w, t)$	Difference	DID: effect of program τ
Program group	$w = 1, t = 0$	$\gamma_0 + \gamma_2$	$\gamma_1 + \gamma_3$	γ_1
	$w = 1, t = 1$	$\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3$		
Control group	$w = 0, t = 0$	γ_0	γ_3	
	$w = 0, t = 1$	$\gamma_0 + \gamma_3$		

2 A simple model of program evaluation

Glewwe et al. (2010) studied an education program in Kenya in late 1990s. The program was aiming at improving education quality of primary schools by rewarding the teachers according to student test scores. The conclusion of the paper, generally speaking, is that the program increased student test scores but there is little evidence that overall education quality was improved. For details, please read "Teacher Incentives", *American Economic Journal: Applied Economics* 2(3): 205-227.

Let's focus on the methods of the paper. The education program was carefully conducted to follow the experimental criteria. 100 schools were randomly divided into the program and control groups. So the ignorability condition is satisfied. Let the binary variable $w = 1$ denote that a kid was attending a program school and $w = 0$ denote that a kid was in a control school. The outcome of program is measured by the test score $y = y_0 + w(y_1 - y_0)$, where y_1 is the score of a kid in the program school and y_0 is the score if this kid instead attended a control school. In the data, of course, each kid has either y_1 or y_0.

The effect of program τ is

$$\tau = \mathrm{E}(y_1 - y_0)$$

Consider the linear regression model $\mathrm{E}(y \mid w) = \gamma_0 + \gamma_1 w$, or

$$y = \gamma_0 + \gamma_1 w + u \tag{1}$$

Because

$$\begin{aligned} \mathrm{E}(y \mid w) &= \mathrm{E}(y_0 + w(y_1 - y_0) \mid w) \\ &= \mathrm{E}(y_0 \mid w) + \mathrm{E}(w(y_1 - y_0) \mid w) \\ &= \mathrm{E}(y_0 \mid w) + w\mathrm{E}(y_1 - y_0 \mid w) \\ &= \mathrm{E}(y_0) + \mathrm{E}(y_1 - y_0)w \end{aligned}$$

we know $\tau = \gamma_1$.

3 Difference in differences again

Now let's think about using DID to estimate the effect of the Kenya education program. Suppose we have the data of the program and control schools in both base year and program year. Then we can run the following linear regression

$$y = \gamma_0 + \gamma_1 wt + \gamma_2 w + \gamma_3 t + u \tag{2}$$

and γ_1 is just the effect of program τ.

However, Glewwe et al. (2010) followed a slightly different approach. They mainly used the data in the program year $t = 1$, which is the same as what they used in model (1). In addition, they used the data of average test scores of the program and control schools in the base year, denoted by $\overline{y}_{base}$.

To see how this works, let's go back to the table in the first section. Notice that in the table γ_2 represents the difference between the program and control groups before the program. Actually, such difference between two groups in the base year is captured by $\overline{y}_{base}$. Let's think about the following model

$$y = \gamma_0 + \gamma_1 w + \beta_2 \overline{y}_{base} + \gamma_3 + u \tag{3}$$

Because $t = 1$ for all the data, (3) is the same as (2). We can rewrite (3) as the following

$$y = \beta_0 + \gamma_1 w + \beta_2 \overline{y}_{base} + u \tag{4}$$

(4) is the DID model used by Glewwe et al. (2010). We see that this model is equivalent to the standard DID model (2), and only slightly different from the basic program evaluation model (1).